# GCSE EDEXCEL MATHS

# Aiming for Grade 9

**REVISION BOOKLET** 



Name:

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## <u>Surds</u>

## Things to remember:

- √ means square root;
- To simplify surds, find all its factors;
- To rationalise the denominator, find an equivalent fraction where the denominator is rational.

#### **Questions:**

1. Work out

$$\frac{(5+\sqrt{3})(5-\sqrt{3})}{\sqrt{22}}$$

Give your answer in its simplest form.

 (Total 3 marks)

**2.** (a) Rationalise the denominator of  $\frac{1}{\sqrt{3}}$ 

(1)

(b) Expand  $(2 + \sqrt{3})(1 + \sqrt{3})$ Give your answer in the form  $a + b\sqrt{3}$  where a and b are integers.

																			(	2	2	)

(Total 3 marks)

3. (a) Rationalise the denominator of  $\frac{1}{\sqrt{7}}$ 

(2)

(b) (i) Expand and simplify  $(\sqrt{3} + \sqrt{15})^2$  Give your answer in the form  $a + b\sqrt{3}$  where a and b are integers.

.....

(ii) All measurements on the triangle are in centimetres.ABC is a right-angled triangle.k is a positive integer.

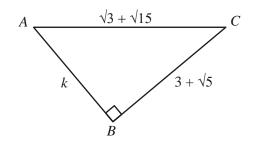


Diagram **NOT** accurately drawn

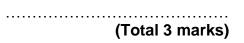
Find the value of k.

k = .....

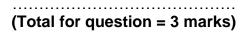
(Total 7 marks)

4.	Expai	and and simplify $(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})$	
5.	(a)	Write down the value of $49^{1/2}$	(Total 2 marks
	(b)	Write $\sqrt{45}$ in the form $k\sqrt{5}$ , where $k$ is an integer.	
6.	Write	$a + \frac{\sqrt{18} + 10}{\sqrt{2}}$ in the form $a + b\sqrt{2}$ where $a$ and $b$ are integers.	(1 (Total 2 marks

7. Expand and simplify  $(2 + \sqrt{3})(7 - \sqrt{3})$ Give your answer in the form  $a + b\sqrt{3}$  where a and b are integers.



8. Rationalise the denominator of  $\frac{(4+\sqrt{2})(4-\sqrt{2})}{\sqrt{7}}$  Give your answer in its simplest form.



9. Show that  $\frac{(4-\sqrt{3})(4+\sqrt{3})}{\sqrt{13}}$  simplifies to  $\sqrt{13}$ 

(Total for question = 2 marks)

## **Algebraic Proofs**

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- Start by expanding the brackets, then factorise.
- Remember the following:
  - 1.  $2n \rightarrow \text{even number}$
  - 2.  $2n + 1 \rightarrow odd number$
  - 3.  $a(bn + c) \rightarrow multiple of a$
  - 4. Consecutive numbers are numbers that appear one after the other.

0		es	ŧi	^	n	c	
w	u	53	u	v		Э	

Expand and simplify x(x + 1)(x - 1)1. (a)

**(2)** 

In a list of three consecutive positive integers at least one of the numbers is even and one of the numbers is a multiple of 3

n is a positive integer greater than 1

Prove that n³ - n is a multiple of 6 for all possible values of n.

(2)

 $2^{61}$  – 1 is a prime number.

Explain why 2<sup>61</sup> + 1 is a multiple of 3

(2)

(Total for question = 6 marks)

2.		e that (2n + 3)² – (2n – 3)² is a multiple of 8 I positive integer values of n.	
3.	(a)	Expand and simplify (y - 2)(y - 5)	(Total for Question is 3 marks)
	*(b)	Prove algebraically that (2n + 1) <sup>2</sup> - (2n + 1) is an even number for all positive integer values of n.	(2)

(3) (Total for Question is 5 marks)

4.	* Pro integ	ve alg	ebraically that the difference between the squ equal to the sum of these two integers.	ares of any two consecutive
_	( )	<i></i>		(Total for Question is 4 marks)
5.	(a)	Fact	orise x² + 7x	
	(b)	Fact	orise y² – 10y + 16	(1)
	*(c)	(i)	Factorise 2t <sup>2</sup> + 5t + 2	(2)
		(ii)	t is a positive whole number. The expression 2t <sup>2</sup> + 5t + 2 can never have Explain why.	
				(3) (Total for Question is 6 marks)

4.

6.	(a)	Factorise 3t + 12

(1)

(b) (i) Expand and simplify 7(2x + 1) + 6(x + 3)

.....

(ii) Show that when x is a whole number 7(2x + 1) + 6(x + 3) is always a multiple of 5

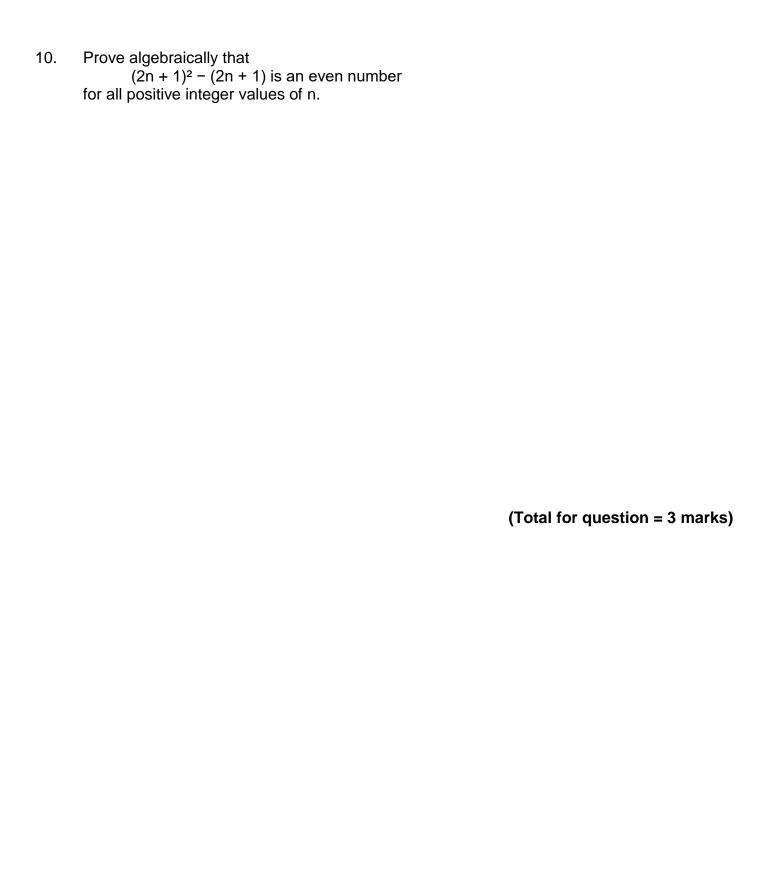
(3)

(Total for Question is 4 marks)

7. Prove that  $(n-1)^2 + n^2 + (n+1)^2 = 3n^2 + 2$ 

(Total for Question is 2 marks)

8.	Prove algebraically that the difference between the squares of any twis equal to the sum of these two integers.	o consecutive integers
	(Total for	<sup>·</sup> question is 4 marks)
9.	The product of two consecutive positive integers is added to the large integers. Prove that the result is always a square number.	er of the two
	(Total fo	r question = 3 marks)
	(Total to	. 44004011 <b>–</b> 0 11141 <b>K</b> 3)



## **Transformations of graphs**

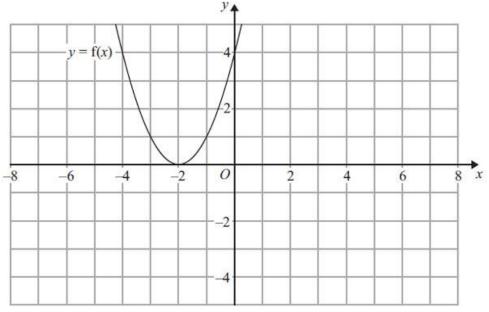
#### Things to remember:

- 1. f(x) means the function of x.
- 2.  $-\dot{f}(x)$  is a reflection in the x-axis.
- 3. f(-x) is a reflection in the y-axis.
- 4. f(x a) is a translation in the x-axis, a units.
- 5. f(x) + b is a translation in the y-axis, b units.
- 6. cf(x) is an enlargement in the y-axis, scale factor c.
- 7. f(dx) is an enlargement in the x-axis, scale factor  $\frac{1}{d}$ .

#### **Questions:**

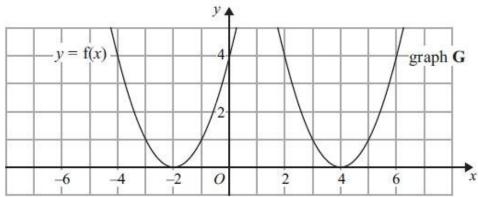
1. y = f(x)

The graph of y = f(x) is shown on the grid.



(a) On the grid above, sketch the graph of y = -f(x).

The graph of y = f(x) is shown on the grid.



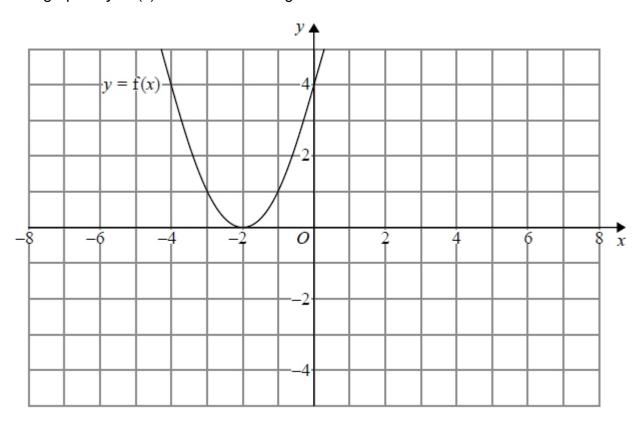
The graph **G** is a translation of the graph of y = f(x).

(b) Write down the equation of graph **G**.

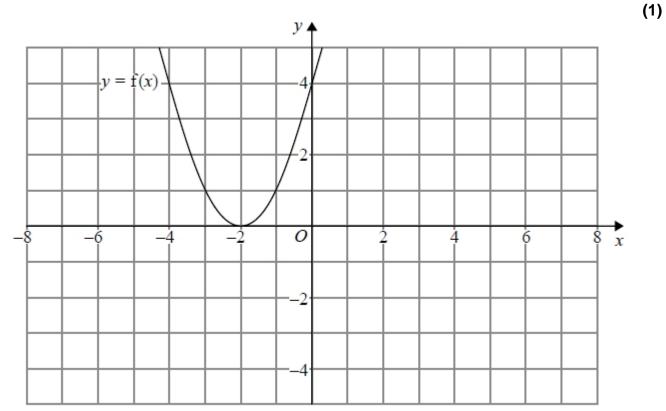
(Total for Question is 3 marks)

(2)

**2.** The graph of y = f(x) is shown on both grids below.



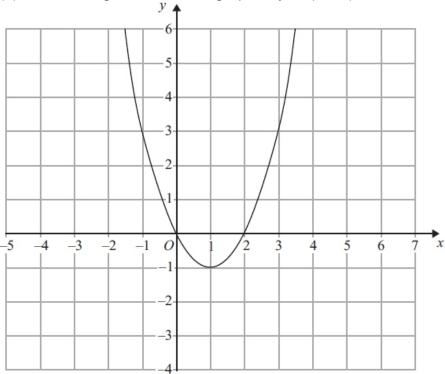
(a) On the grid above, sketch the graph of y = f(-x)



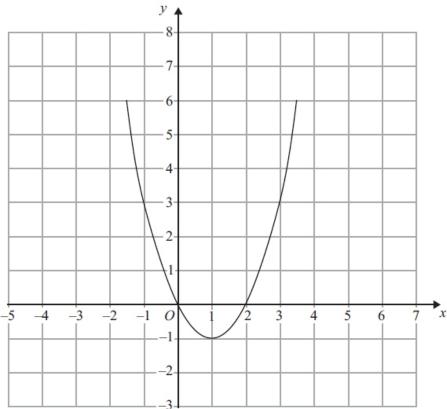
(b) On this grid, sketch the graph of y = -f(x) + 3

(1) (Total for question = 2 marks)

- 3.
- The graph of y = f(x) is shown on each of the grids. (a) On this grid, sketch the graph of y = f(x 3)



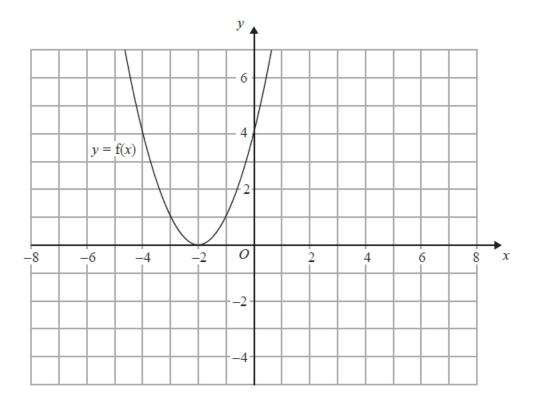
On this grid, sketch the graph of y = 2f(x)(b)



(Total for Question is 4 marks)

(2)

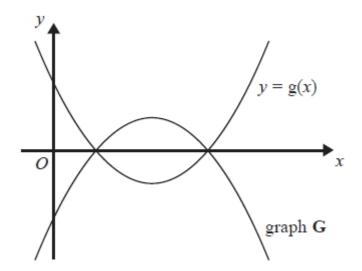
The graph of y = f(x) is shown on the grid. 4.



On the grid above, sketch the graph of y = f(x + 3)(a)

(2)

The graph of y = g(x) is shown below.



The graph **G** is the reflection of y = g(x) in the *x*-axis. (b) Write down an equation of graph **G**.

(1)

(Total for question = 3 marks)

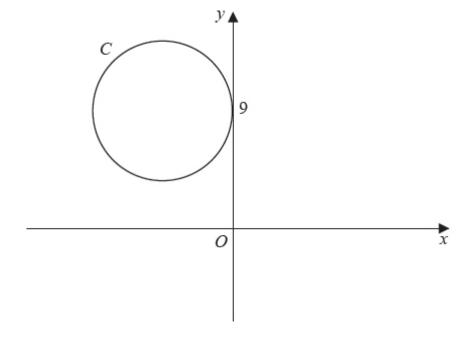
## **Equations of Circles**

#### Things to remember:

- 8. The general equation of a circle is  $(x a)^2 + (y b)^2 = r^2$ , where (a, b) is the centre and r is the radius.
- 9. To calculate the equation of the tangent:
  - 1. Calculate the gradient of the radius of the circle.
  - 2. Calculate the gradient of the tangent of the circle.
  - 3. Substitute the given coordinate and the gradient of the tangent into y = mx + c to calculate the y-intercept.

#### **Questions:**

- 1. The circle C has radius 5 and touches the y-axis at the point (0, 9), as shown in the diagram.
  - (a) Write down an equation for the circle C, that is shown in the diagram.



(3)

A line through the point P(8, -7) is a tangent to the circle C at the point T.

(b) Find the length of PT.

(3) (Total 6 marks)

							(Total 9 ma	(5) rks
	(=)							
	(b)	Find an e	equation for t	he tangent to	C at P.			(4)
3.	The o	circle C has Find an e	s centre (3, 1 equation for (	) and passes C.	through the	e point P(8, 3).	(Total 4 ma	rks)
۷.			( , ,	•	3	, ,	an equation for C.	

## **Quadratic and Other Sequences**

	Th	ind	as	to	rem	em	bei	۲:
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•		
_	<ol> <li>To calculate the nth term of a quadratic sequence:</li> <li>Calculate the first difference.</li> <li>Calculate the second difference.</li> <li>How many n²s?</li> <li>Subtract.</li> <li>Calculate the nth term of the difference.</li> <li>Write the quadratic nth term.</li> </ol>	
	·	
ર્ગues I.	stions:  Here are the first 5 terms of a quadratic sequence.  1 3 7 13 21	
	Find an expression, in terms of <i>n</i> , for the <i>n</i> th term of this	quadratic sequence.
		(Total for question is 3 marks)
2.	Here are the first six terms of a Fibonacci sequence.  1 1 2 3 5 8  The rule to continue a Fibonacci sequence is, the next term in the sequence is the sum of the two (a) Find the 9th term of this sequence.	(Total for question is 3 marks) o previous terms.
2.	1 1 2 3 5 8  The rule to continue a Fibonacci sequence is, the next term in the sequence is the sum of the two	o previous terms. (1)

	en that the 3rd term is 7 and the 6th term find the value of <i>a</i> and the value of <i>b</i> .	
a =		
b =		
(3) (Total for question = 6 marks)		
32 50	e are the first five terms of a sequence.  2 8 18 32 50  Find the next term of this sequence.	3.
	nth term of a different sequence is 3.6 Work out the 5th term of this sequence	
(1) (Total for question = 2 marks		

•	Here	are the fir 1 5	s of an arithmet 13 17	ic sequence.			
	(a)	_	ession, in terms	of <i>n</i> , for the <i>r</i>	oth term of thi	is sequence.	
				•			(2)
	The (b)		number sequer of this sequence				
					(Total for	r Question is 4	(2)   marks

## **Completing the Square**

#### Things to remember:

- To complete the square:
  - 1. Halve the coefficient of x.
  - 2. Put in brackets with the x and square the brackets.
  - 3. Subtract the half-coefficient squared.
  - 4. Don't forget the constant on the end!
  - 5. Simply.
- For  $(x p)^2 + q = 0$ , the turning point is (p, q).

#### **Questions:**

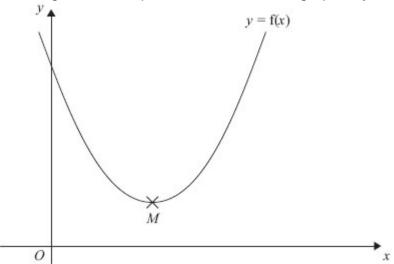
1. (i) Sketch the graph of  $f(x) = x^2 - 5x + 10$ , showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.

(ii) Hence, or otherwise, determine whether f(x + 2) - 3 = 0 has any real roots. Give reasons for your answer.

(Total for question = 6 marks)

2.	(a)	Write $2x^2 + 16x + 35$ in the form $a(x + b)^2 + c$ where $a$ , $b$ , and $c$ are integers.
	(b)	Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = 2x^2 + 16x + 35$
		<b>(1</b> )
		(Total for question = 4 marks)
3.	The e (a)	expression $x^2 - 8x + 21$ can be written in the form $(x - a)^2 + b$ for all values of $x$ . Find the value of $a$ and the value of $b$ .
		a =
		b =(3)

The equation of a curve is y = f(x) where  $f(x) = x^2 - 8x + 21$ The diagram shows part of a sketch of the graph of y = f(x).



The minimum point of the curve is *M*. (b) Write down the coordinates of *M*.

(1)

(Total for Question is 4 marks)

## **Inverse and Composite Functions**

#### Things to remember:

- y = f(x) means that y is a function of x.
- f(a) means the value of x is a, so substitute x with a.
- The graph of the inverse is the reflection of the graph in the line y = x
- We find the inverse function by putting the original function equal to y and rearranging to make x the subject.
- We use the notation  $f^{-1}(x)$  for the inverse function.
- When a function is followed by another, the result is a composite function.
- fg(x) means do g first, followed by f.

#### **Questions:**

1. The functions f and g are such that

$$f(x) = 1 - 5x$$
 and  $g(x) = 1 + 5x$ 

(a) Show that gf(1) = -19

(b) Prove that 
$$f^{-1}(x) + g^{-1}(x) = 0$$
 for all values of  $x$ .

**(2)** 

2.	The fu	unction f is such that $f(x) = 4x - 1$ Find $f^{-1}(x)$	
	Given	unction g is such that $g(x) = kx^2$ where $k$ is a constant. In that $fg(2) = 12$ work out the value of $k$	$f^{-1}(x) = \dots$ (2)
3.	The fu	unctions f and g are such that $f(x) = 3(x - 4) \text{ and } g(x) = \frac{x}{5} + 1$ Find the value of f(10)	k =(2) (Total for question = 4 marks)
	(b)	Find $g^{-1}(x)$	(1)

$$g^{-1}(x) = \dots$$
 (2)

(c) Show that ff(x) = 9x - 48

(2) (Total for question = 5 marks)

4.  $f(x) = 3x^2 - 2x - 8$ <br/>Express f(x + 2) in the form  $ax^2 + bx$ 

(Total for question is 3 marks)

## **Expanding more than two binomials**

## Things to remember:

- Start by expanding two pair of brackets using the grid or FOIL method.
- Then expand the third set of brackets.
- Use columns to keep x3, x2 etc in line to help with addition.

<u> </u>		-4	٠.		
w	ue	sti	Ю	ทร	5:

1	Show the	٠+
	31 IU VVV 11 I	11

$$(x-1)(x+2)(x-4) = x^3 - 3x^2 - 6x + 8$$
 for all values of x.

	or qu		mar	

2. Show that

$$(3x - 1)(x + 5)(4x - 3) = 12x^3 + 47x^2 - 62x + 15$$
 for all values of x.

(Total for question is 3 marks)

3.	Show that
	$(x-3)(2x+1)(x+3) = 2x^3 + x^2 - 18x - 9$
	for all values of x.

(Total	for que	estion	is 3	marks)

4.  $(2x + 1)(x + 6)(x - 4) = 2x^3 + ax^2 + bx - 24$  for all values of x, where a and b are integers. Calculate the values of a and b.

## **Nonlinear Simultaneous Equations**

#### Things to remember:

- 1. Substitute the linear equation into the nonlinear equation.
- 2. Rearrange so it equals 0.
- 3. Factorise and solve for the first variable (remember there will be two solutions).
- 4. Substitute the first solutions to solve for the second variable.
- 5. Express the solution as a pair of coordinate where the graphs intersect.

#### **Questions:**

1.	Solve the equations
	$x^2 + y^2 = 36$
	x = 2y + 6

(Total for Question is 5 marks)

3. Solve the simultaneous equations  $x^2 + y^2 = 25$  y = 2x + 5

x = and $y =$
---------------

4. Solve the simultaneous equations  $x^2 + y^2 = 9$ x + y = 2

Give your answers correct to 2 decimal places.

 $x = \dots y = \dots y = \dots$  or  $x = \dots y = \dots y = \dots$  (Total for Question is 6 marks)

5.	Solve algebraically the simultaneous equations $x^2 + y^2 = 25$ y - 2x = 5	
	y-2x=5	
		/Total for Organian in Francis -
		(Total for Question is 5 marks)

## **Solving Quadratic Inequalities**

## Things to remember:

Start by solving the quadratic to find the values of x, then sketch the graph to determine the

**Questions:** 

Solve

$$x^2 > 3x + 4$$

- (Total for question = 3 marks)
- Solve the inequality  $x^2 > 3(x + 6)$ 2.

$$x^2 > 3(x+6)$$

(Total for question = 4 marks)

3. Solve the inequality

$$x^2 + 5x > 6$$

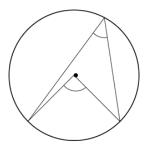
(Total for question = 3 marks)

4.	Solve the inequality	X <sup>2</sup> - 2X + 8 < 0	
5.	Solve the inequality	x² - x ≥ 12	(Total for question = 3 marks)
6.	Solve the inequality	$x^2 \le 4(2x + 5)$	(Total for question = 3 marks)

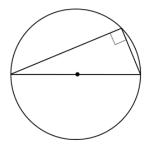
(Total for question = 4 marks)

## **Circle theorems**

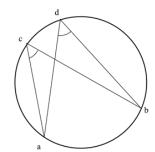
## Things to remember:



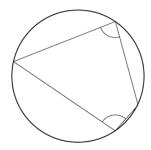
The angle at the centre is twice the angle at the circumference.



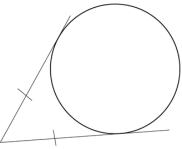
The angle in a semicircle is 90°.



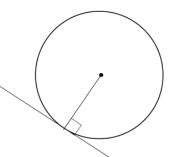
Angles subtended by the same arc are equal.



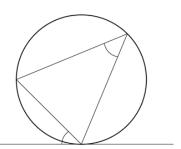
Opposite angles in a cyclic quadrilateral sum to 180°.



Tangents from a point are equal.



A tangent is perpendicular to a radius.



Angles in alternate segments are equal.

#### **Questions:**

1.

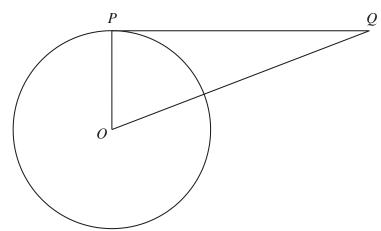


Diagram **NOT** accurately drawn

*P* is a point on the circumference of the circle, centre *O*.

PQ is a tangent to the circle.

(i) Write down the size of angle *OPQ*.

•	•	•	•	•	•	•	•	•	٠	٠	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	٠	•	•	•	•	•

(ii) Give a reason for your answer.


(Total 2 marks)

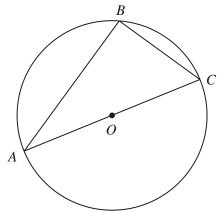


Diagram NOT accurately drawn

A, B and C are points on the circumference of a circle, centre O.

AC is a diameter of the circle.

	(ii)	Give a reason for your answer.	••
(a)	(i)	Write down the size of angle ABC.	

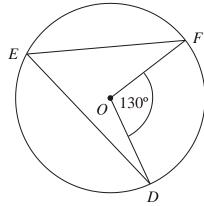


Diagram **NOT** accurately drawn

D,  $\vec{E}$  and  $\vec{F}$  are points on the circumference of a circle, centre O. Angle  $DOF = 130^{\circ}$ .

- (b) (i) Work out the size of angle *DEF*.
  - (ii) Give a reason for your answer.

(2) (Total 4 marks)

3.

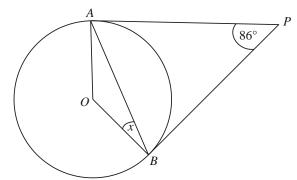


Diagram NOT accurately drawn

A and B are points on the circumference of a circle, centre O.

PA and PB are tangents to the circle.

Angle APB is 86°.

Work out the size of the angle marked *x*.

° (Total 2 marks)

4.

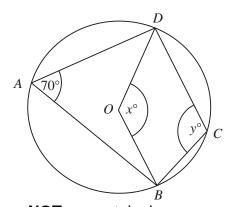


Diagram **NOT** accurately drawn

In the diagram, A, B, C and D are points on the circumference of a circle, centre O.

Angle  $BAD = 70^{\circ}$ .

Angle  $BOD = x^{\circ}$ .

Angle  $BCD = y^{\circ}$ .

- (a) (i) Work out the value of x.
  - (ii) Give a reason for your answer.

(2)

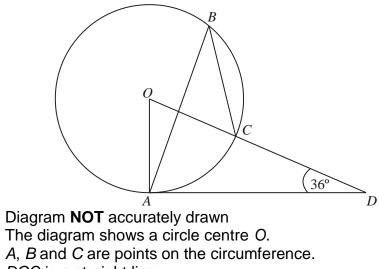
(b) (i) Work out the value of *y*.

(ii) Give a reason for your answer.

....(2)

(Total 4 marks)

5.



DCO is a straight line.

DA is a tangent to the circle. Angle  $ADO = 36^{\circ}$ 

(a)	Work out the size of	angle <i>AOD</i> .

(2)
c
(3) tal 5 marks)

#### **Vectors**

### Things to remember:

- Use the letter provided in the question.
- Going against the arrow is a negative.
- Vectors need to be written in bold or underlined.
- They can be manipulated similarly to algebra.

#### **Questions:**

**1.** The diagram shows a regular hexagon *ABCDEF* with centre *O*.

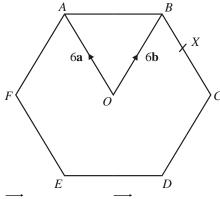


Diagram **NOT** accurately drawn

 $\overrightarrow{OA} = 6\mathbf{a}$   $\overrightarrow{OB} = 6\mathbf{b}$ 

(a) Express in terms of **a** and/or **b** 

(i)  $\overrightarrow{AB}$ ,

(ii)  $\overrightarrow{EF}$ .

.....

.....

(2)

X is the midpoint of BC.

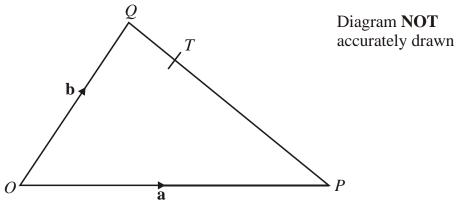
(b) Express  $\overrightarrow{EX}$  in terms of **a** and/or **b** 

(2)

Y is the point on AB extended, such that AB: BY = 3:2

(c) Prove that E, X and Y lie on the same straight line.

(3) (Total 7 marks) 2. T is the point on PQ for which PT: TQ = 2:1.



OPQ is a triangle.

$$\overrightarrow{OP} = \mathbf{a}$$
 and  $\overrightarrow{OQ} = \mathbf{b}$ .

(a) Write down, in terms of  $\bf a$  and  $\bf b$ , an expression for  $\overrightarrow{PQ}$ .

$\overrightarrow{PQ}$	=	 	 	 	 		 		 			 			 		 					 				
																						ĺ	(	1	ı	١

(b) Express  $\overrightarrow{OT}$  in terms of **a** and **b**. Give your answer in its simplest form.

$$\overrightarrow{OT}$$
 = ......(2) (Total 3 marks)

**3.** *OABC* is a parallelogram.

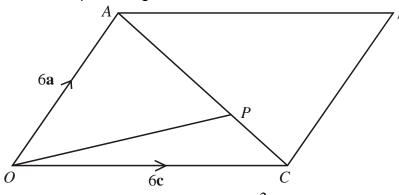


Diagram **NOT** accurately drawn

*P* is the point on *AC* such that  $AP = \frac{2}{3}AC$ .

$$\overrightarrow{OA} = 6a$$
.  $\overrightarrow{OC} = 6c$ .

(a) Find the vector  $\overrightarrow{OP}$ . Give your answer in terms of **a** and **c**.

	(3)

The midpoint of CB is M.

(b) Prove that *OPM* is a straight line.

(2) (Total 5 marks) OPQ is a triangle.R is the midpoint of OP.S is the midpoint of PQ.

$$\overrightarrow{OP} = \mathbf{p} \text{ and } \overrightarrow{OQ} = \mathbf{q}$$

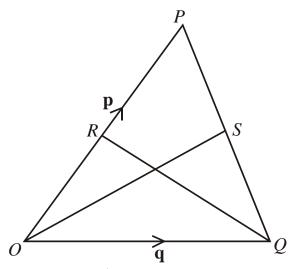


Diagram **NOT** accurately drawn

(i) Find  $\overrightarrow{OS}$  in terms of **p** and **q**.

$\overrightarrow{OS}$																				
Ob	=	 	 ٠.				 	 	 ٠.	 	 ٠.					 	 			٠.

(ii) Show that RS is parallel to OQ.

(Total 5 marks)

OPQR is a trapezium with PQ parallel to OR. 5.

$$\overrightarrow{OP}$$
 = 2**b**

$$\overrightarrow{PQ}$$
 = 2a  $\overrightarrow{OR}$  = 6a

$$\overrightarrow{OR} = 66$$

M is the midpoint of PQ and N is the midpoint of OR.

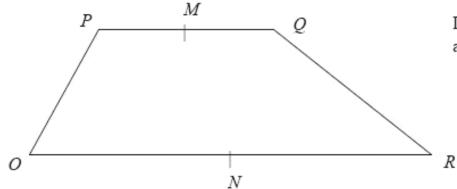


Diagram NOT accurately drawn

Find the vector  $\overrightarrow{MN}$  in terms of **a** and **b**. (a)

$\overrightarrow{MN}$	=	 													
													(	(2	2

X is the midpoint of MN and Y is the midpoint of QR.

Prove that XY is parallel to OR. (b)

(Total 4 marks)

#### ABCD is a straight line. 6.

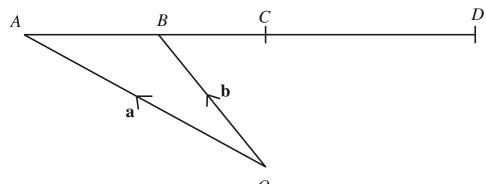


Diagram **NOT** accurately drawn

O is a point so that  $\overrightarrow{OA} = \mathbf{a}$  and  $\overrightarrow{OB} = \mathbf{b}$ .

B is the midpoint of AC.

C is the midpoint of AD.

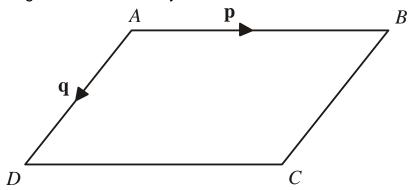
Express, in terms of  $\mathbf{a}$  and  $\mathbf{b}$ , the vectors

 $\overrightarrow{AC}$ (i)

(ii)	$\overrightarrow{OD}$
(11)	$\longrightarrow$

|--|

7. Diagram **NOT** accurately drawn



ABCD is a parallelogram. AB is parallel to DC.

AD is parallel to BC.

$$\stackrel{
ightarrow}{AB}={\bf p}$$

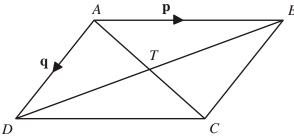
$$\stackrel{\rightarrow}{AD}$$
 = q

 $\dot{E}$ xpress, in terms of p and q (a)

- $\overrightarrow{AC}$ (i)
- $\stackrel{
  ightarrow}{BD}$ (ii)

**(2)** 

Diagram **NOT** accurately drawn



AC and BD are diagonals of parallelogram ABCD. AC and BD intersect at T.

Express  $\overrightarrow{AT}$  in terms of **p** and **q**. (b)

(1)	
(Total 3 marks)	

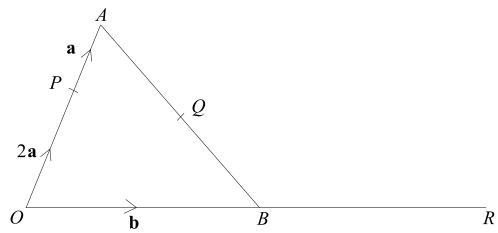
8. Diagram **NOT** accurately drawn

OAB is a triangle.

B is the midpoint of OR.

Q is the midpoint of AB.

$$\overrightarrow{OP} = 2\mathbf{a} \quad \overrightarrow{PA} = \mathbf{a} \quad \overrightarrow{OB} = \mathbf{b}$$



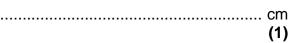
- (a) Find, in terms of a and b, the vectors
  - AB. (i)
  - $\overrightarrow{PR}$ (ii)

(iii) $PQ$ .	
	(4)
 Harris Hala I. BOD's a codal d'a	(7)

(b) Hence explain why *PQR* is a straight line.

	(2)
The length of PQ is 3 cm.	

Find the length of PR. (c)



(Total 7 marks)

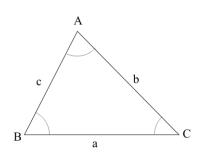
# **Sine and Cosine Rules**

## Things to remember:

• For any triangle ABC,  $a^2 = b^2 + c^2 - 2bc \cos A$ 

• For any triangle ABC,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ 

For any triangle ABC, area = ½ a b sinC



#### **Questions:**

1. Diagram **NOT** accurately drawn

ABC is a triangle.

D is a point on AC.

Angle  $BAD = 45^{\circ}$ 

Angle  $ADB = 80^{\circ}$ 

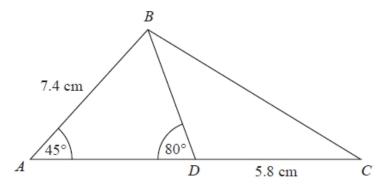
AB = 7.4 cm

DC = 5.8 cm

Work out the length of *BC*.

Give your answer correct to 3

significant figures.



2. Diagram **NOT** accurately drawn

ABC is a triangle.

AB = 8.7 cm.

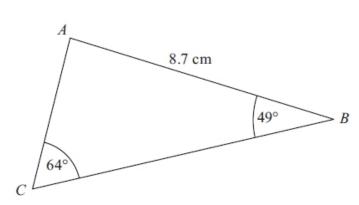
Angle  $ABC = 49^{\circ}$ .

Angle  $ACB = 64^{\circ}$ .

Calculate the area of triangle ABC.

Give your answer correct to 3 significant

figures.



......cm<sup>2</sup>
(Total for Question is 5 marks)

ABCD is a quadrilateral.
 Diagram NOT accurately drawn
 Work out the length of DC.
 Give your answer correct to 3 significant figures.

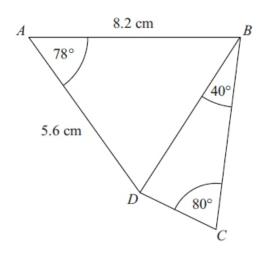
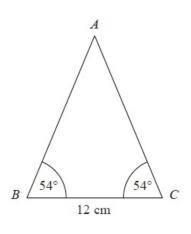
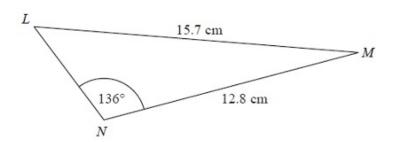


Diagram NOT accurately drawn
 ABC is an isosceles triangle.
 Work out the area of the triangle.
 Give your answer correct to 3 significant figures.

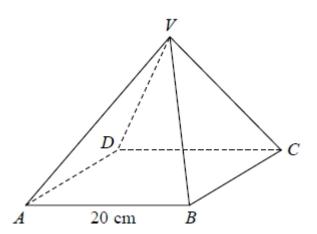


......cm<sup>2</sup>
(Total for Question is 4 marks)

5. Diagram NOT accurately drawn The diagram shows triangle LMN. Calculate the length of LN. Give your answer correct to 3 significant figures.



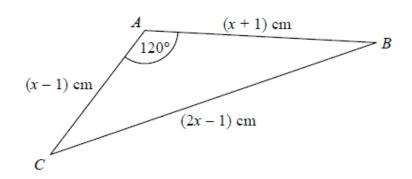
VABCD is a solid pyramid.
 ABCD is a square of side 20 cm.
 The angle between any sloping edge and the plane ABCD is 55°
 Calculate the surface area of the pyramid.
 Give your answer correct to 2 significant figures.



(Total for Question is 5 marks)

7. The diagram shows triangle ABC. The area of triangle ABC is  $k\sqrt{3}$  cm<sup>2</sup>.

Find the exact value of *k*.



k	=	 																		
			(	To	ta	ıl :	foi	r	qι	ıe	st	tic	n	=	7	n	na	arl	ks	;)

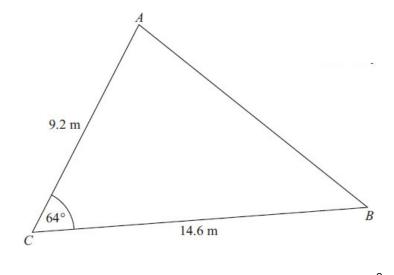
8. Diagram **NOT** accurately drawn

AC = 9.2 m

BC = 14.6 m

Angle ACB = 64°

(a) Calculate the area of the triangle *ABC*.
Give your answer correct to 3 significant figures.



..... m² **(2)** 

(b) Calculate the length of *AB*. Give your answer correct to 3 significant figures.

																									1	3	Ł	١

(Total for Question is 5 marks)

# **Area under Graphs**

#### Things to remember:

- Velocity is speed with direction
- Acceleration and deceleration is given by the gradient of the graph  $\left(\frac{rise}{run}\right)$
- The distance travelled is given by the area under the graph.

#### **Questions:**

1. A car has an initial speed of *u* m/s.

The car accelerates to a speed of 2u m/s in 12 seconds.

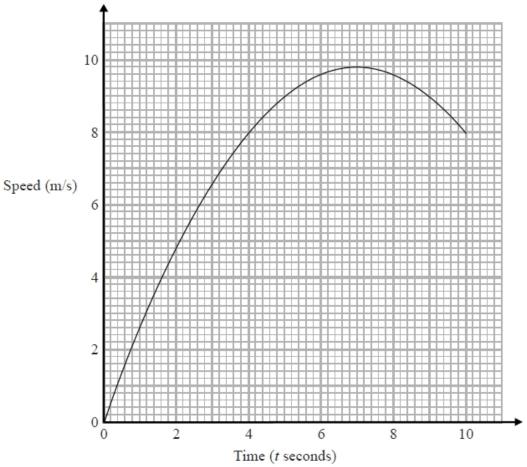
The car then travels at a constant speed of 2u m/s for 10 seconds.

Assuming that the acceleration is constant, show that the total distance, in metres, travelled by the car is 38u.

(Total for question = 4 marks)

2. Karol runs in a race.

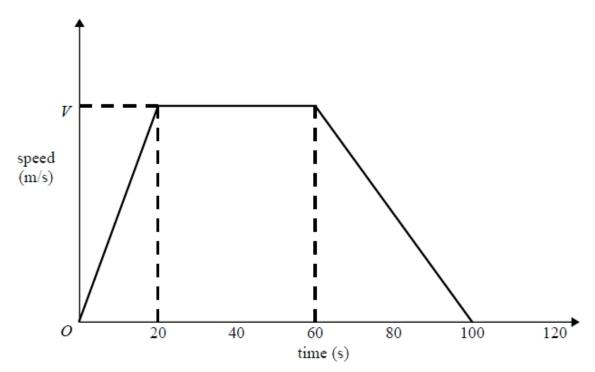
The graph shows her speed, in metres per second, *t* seconds after the start of the race.



(a) Calculate an estimate for the gradient of the graph when t = 4 You must show how you get your answer.

(3)	Describe fully what your answer to part (a) represents.	(b)
(2)	Explain why your answer to part (a) is only an estimate.	(c)
(1) otal for question = 6 marks)	(Т	

3. Here is a speed-time graph for a car journey. The journey took 100 seconds.

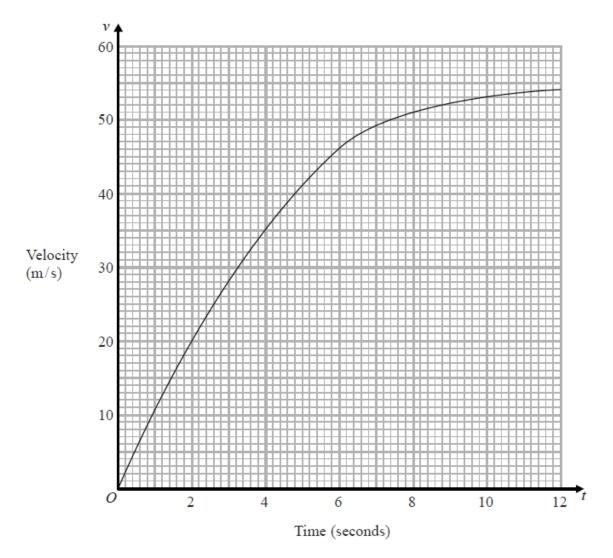


The car travelled 1.75km in the 100 seconds.

(a) Work out the value of V.

(3)		/L \
	Describe the acceleration of the car for each part of this journey.	(b)
(2) uestion = 5 marks)	(Total for qu	

4. The graph shows information about the velocity, *v* m/s, of a parachutist *t* seconds after leaving a plane.



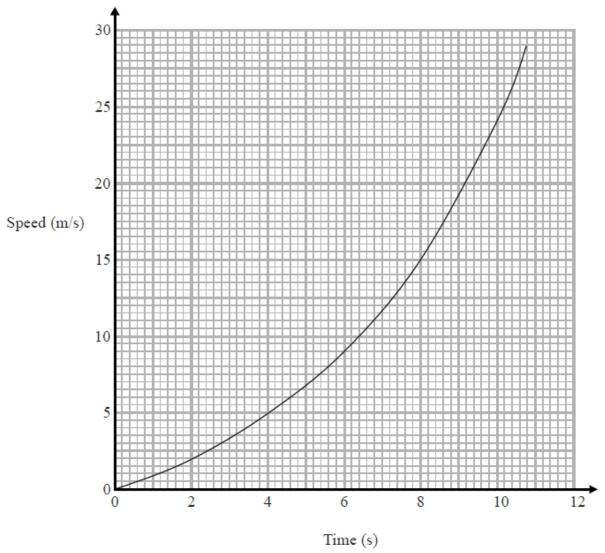
(a) Work out an estimate for the acceleration of the parachutist at t = 6

	m/s <sup>2</sup>
	(2)

(b) Work out an estimate for the distance fallen by the parachutist in the first 12 seconds after leaving the plane. Use 3 strips of equal width.

m	
(3)	
(Total for augustion is E marks)	

5. Here is a speed-time graph for a car.



(a) Work out an estimate for the distance the car travelled in the first 10 seconds. Use 5 strips of equal width.

	) )
Is your answer to (a) an underestimate or an overestimate of the actual distance?  Give a reason for your answer.	,
(1 (Total for question = 4 marks	)

# **Histograms**

#### Things to remember:

- Frequency = Frequency Density x Class Width;
- The y-axis will always be labelled "frequency density";
- The x-axis will have a continuous scale.

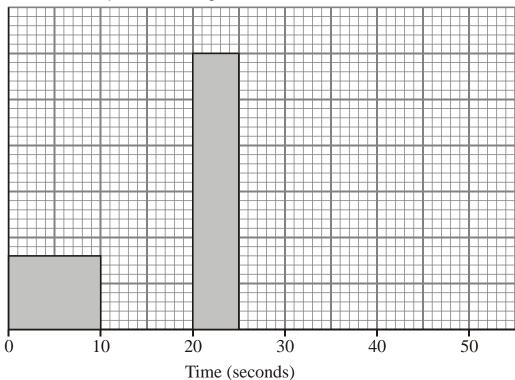
#### **Questions:**

1. One Monday, Victoria measured the time, in seconds, that individual birds spent on her bird table. She used this information to complete the frequency table.

Time (t seconds)	Frequency
0 < <i>t</i> ≤ 10	8
10 < t ≤ 20	16
20 < t ≤ 25	15
25 < t ≤ 30	12
30 < <i>t</i> ≤ 50	6

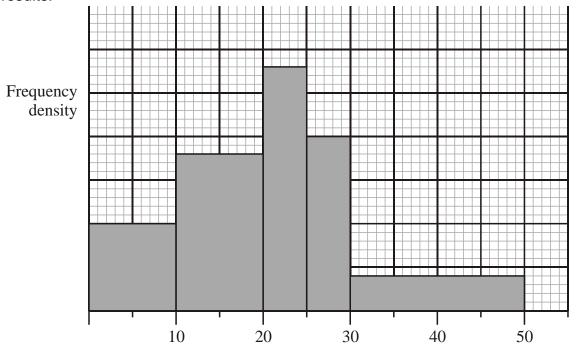
(a) Use the table to complete the histogram.





(3)

On Tuesday she conducted a similar survey and drew the following histogram from her results.



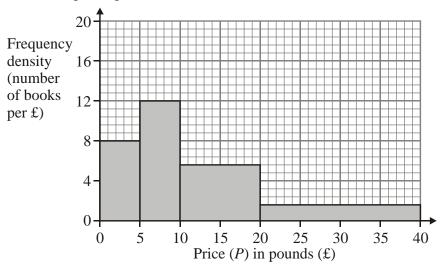
Time (Seconds)

(b) Use the histogram for Tuesday to complete the table.

Time (t seconds)	Frequency
0 < <i>t</i> ≤ 10	10
10 < <i>t</i> ≤ 20	
20 < t ≤ 25	
25 < t ≤ 30	
30 < <i>t</i> ≤ 50	

(2) (Total 5 marks)

This histogram gives information about the books sold in a bookshop one Saturday. 2.



(a) Use the histogram to complete the table.

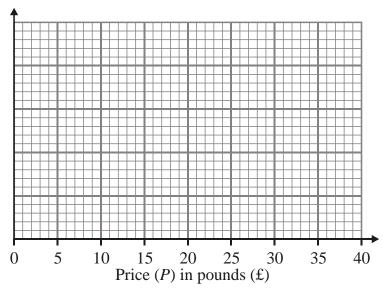
Price (P) in pounds (£)	Frequency
$0 < P \le 5$	
5 < <i>P</i> ≤ 10	
10 < <i>P</i> ≤ 20	
20 < <i>P</i> ≤ 40	

(2)

The frequency table below gives information about the books sold in a second bookshop on the same Saturday.

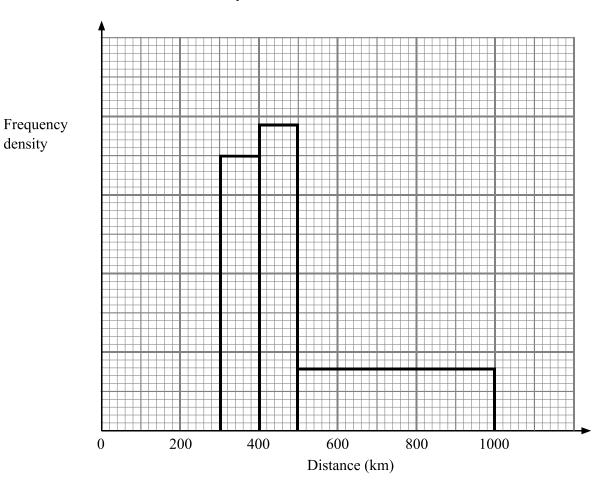
Price (P) in pounds (£)	Frequency
0 < <i>P</i> ≤ 5	80
5 < <i>P</i> ≤ 10	20
10 < <i>P</i> ≤ 20	24
20 < <i>P</i> ≤ 40	96

(b) On the grid below, draw a histogram to represent the information about the books sold in the second bookshop.



(Total 5 marks)

**3.** The incomplete table and histogram give some information about the distances walked by some students in a school in one year.



(a) Use the information in the histogram to complete the frequency table.

Distance (d) in km	Frequency
0 < <i>d</i> ≤ 300	210
300 < <i>d</i> ≤ 400	350
400 < <i>d</i> ≤ 500	
500 < <i>d</i> ≤ 1000	

(2)

(b) Use the information in the table to complete the histogram.

(1)

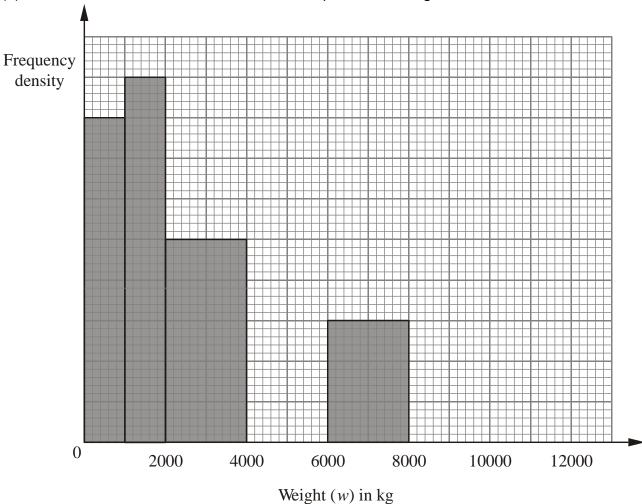
(Total 3 marks)

**4.** The incomplete histogram and table show information about the weights of some containers.

Weight (w) in kg	Frequenc y
$0 < w \le 1000$	16
1000 < <i>w</i> ≤ 2000	
2000 < <i>w</i> ≤ 4000	
4000 < <i>w</i> ≤ 6000	16
6000 < <i>w</i> ≤ 8000	
8000 < <i>w</i> ≤ 12000	8

(a) Use the information in the histogram to complete the table.

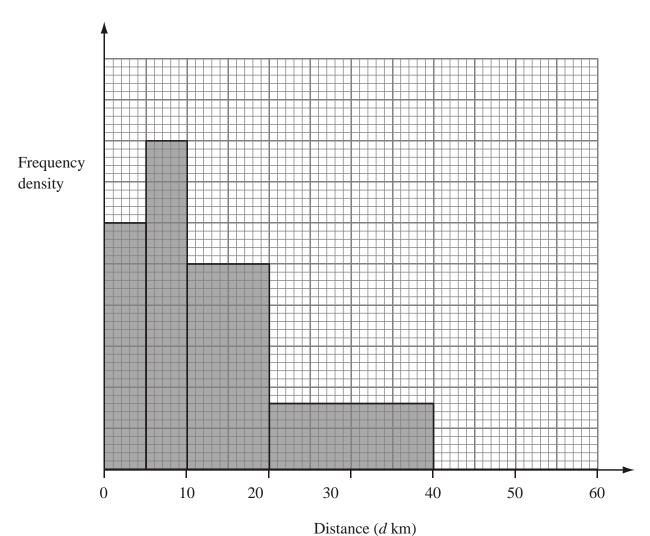
(b) Use the information in the table to complete the histogram.



(Total 4 marks)

(2)

**5.** The incomplete histogram and table give some information about the distances some teachers travel to school.



(a) Use the information in the histogram to complete the frequency table.

Distance (dkm)	Frequency
0 < <i>d</i> ≤ 5	15
5 < <i>d</i> ≤ 10	20
10 < <i>d</i> ≤ 20	
20 < <i>d</i> ≤ 40	
40 < <i>d</i> ≤ 60	10

(b) Use the information in the table to complete the histogram.

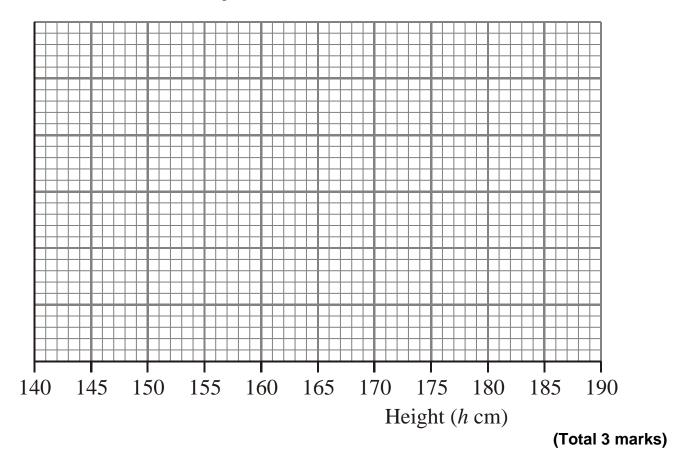
(1) (Total 3 marks)

(2)

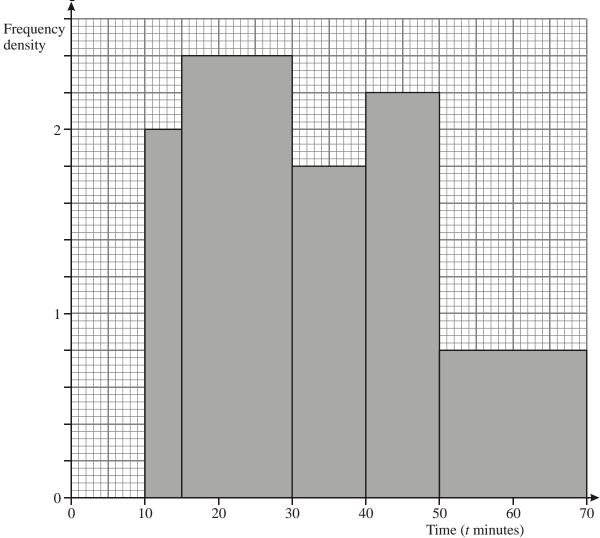
**6.** The table gives information about the heights, in centimetres, of some 15 year old students.

Height (h cm)	145 < <i>h</i> ≤ 155	155 < <i>h</i> ≤ 175	175 < <i>h</i> ≤ 190
Frequency	10	80	24

Use the table to draw a histogram.



**7.** A teacher asked some year 10 students how long they spent doing homework each night. The histogram was drawn from this information.



Use the histogram to complete the table.

Time ( <i>t</i> minutes)	Frequency
10 ≤ <i>t</i> < 15	10
15 ≤ <i>t</i> < 30	
30 ≤ <i>t</i> < 40	
40 ≤ <i>t</i> < 50	
50 ≤ <i>t</i> < 70	

(Total 2 marks)

## **Moving Averages**

#### Things to remember:

- In this context, averages means the mean (add the numbers and divide by how many there were.
- Moving averages are used to identify trends in data peaks, troughs, increasing and decreasing trends.

#### **Questions:**

**1.** The table shows the number of computer games sold in a supermarket each month from January to June.

Jan	Feb	Mar	Apr	May	Jun
147	161	238	135	167	250

Work out the three month moving averages for this information.

	(Total 2	marks)

2. The table shows the number of digital cameras Bytes sold each month in the first six months of 2005.

Month	January	February	March	April	May	June
Number of digital	30	19	20	15	27	39
cameras sold				.0		

The first 3-month moving average for this data is 23

Work out the **second** 3-month moving average for this data.

(Total	2 marks)

3. The table shows the number of orders received each month by a small company.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Number of								
orders	23	31	15	11	19	16	20	13
received								

Work out the first two 4-month moving averages for this data.

and
(Total 3 marks)

4.

5.

A shop sells DVD players.
The table shows the number of DVD players sold in every three-month period from January 2003 to June 2004.

Year	Months	Number of DVD players sold
2003	Jan – Mar	58
	Apr – Jun	64
	Jul – Sep	86
	Oct – Dec	104
2004	Jan – Mar	65
	Apr – Jun	70

(a)	Calculate the	Apr –		moving	average	es for th	s data.			
(b)	What do you players?	r moving	average	es in par	t (a) tell	you abo	out the tre	end in t	he sale of	<b>(2</b> DVD
									(Total 3 m	(1) narks
Montl Montl	able shows the half takings (£) out the 3-poir	Jan 9375	Feb 8907	March 9255	April 9420		months.			
									 (Total 2 m	narks)

6.	The owner of a music shop recorded the number of CDs sold every 3 months. The table shows his records from January 2004 to June 2005.				
	Year Months Number of CDs				

Year	Months	Number of CDs
2004	Jan – Mar	270
	Apr – Jun	324
	Jul – Sept	300
	Oct – Dec	258
2005	Jan – Mar	309
	Apr – Jun	335

(0)	Calculate the	complete set of	of faur paint maying ava	ragge for this information
(a)	Calculate the	complete set c	n tour-point moving ave	rages for this information.

	noving averages suggest?	o) What trend	(b)
(1 (Total 3 marks			

**7.** The table shows some information about student absences.

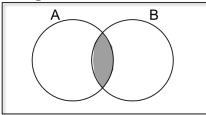
Term	Autumn 2003		Summer 2004			Summer 2005
Number of absences	408	543	351	435	582	372

Work out the three-point moving averages for this information. The first two have been done for you.

434, 443,	
, ,	(Total 2 marks)

# **Set Theory**

## Things to remember:

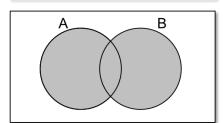


The intersection is where two sets overlap.

$$A \cap B$$

This means A and B.



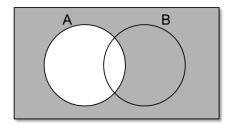


If you put two sets together, you get the union.

$$A \cup B$$

This means **A** or **B**.





The **complement of A** is the region that is not A.

A'

This means **not A**.

#### **Questions:**

1.

$$\mathcal{E}$$
= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

$$A = \{\text{multiples of 2}\}\$$

$$A \cap B = \{2, 6\}$$

$$A \cup B = \{1, 2, 3, 4, 6, 8, 9, 10\}$$

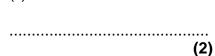
Draw a Venn diagram for this information.

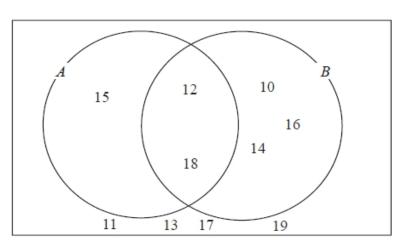
(Total for question is 4 marks)

- 2. Here is a Venn diagram.
  - (a) Write down the numbers that are in set
    - (i) *A* ∪ *B*

.....

(ii)  $A \cap B$ 





One of the numbers in the diagram is chosen at random.

(b) Find the probability that the number is in set A'

(2) (Total for question = 4 marks)

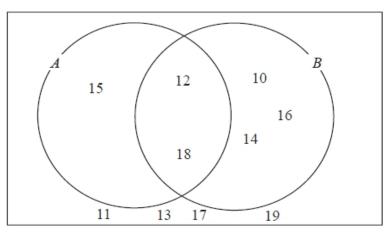
3. Here is a Venn diagram.

- (a) Write down the numbers that are in set
  - (i)  $A \cup B$

.....

(ii)  $A \cap B$ 

(2)



One of the numbers in the diagram is chosen at random.

(b) Find the probability that the number is in set A'

(2)

(Total for question = 4 marks)

All 5 19 p 16 p 21 p 24 p 40 p 1 pe	i asked 50 people which drinks they liked from tea, coffee and milk.  O people like at least one of the drinks eople like all three drinks. eople like tea and coffee but do not like milk. eople like coffee and milk. eople like tea and milk. eople like tooffee. rson likes only milk. i selects at random one of the 50 people. Work out the probability that this person likes tea.	
(a)	Work out the probability that this person likes tea.	
(b)	Given that the person selected at random from the 50 people likes tea, find the probability that this person also likes exactly one other drink.	 •)
	(Total for question = 6 marks	 <u>?</u> ) \$)

# **Proportion**

#### Things to remember:

- Start by checking the question for squares, cubes and roots;
- "x is directly proportional to y" looks like x α y or x = ky
- "x is inversely proportional to y" looks like  $\mathbf{x} \propto \frac{1}{y}$  or  $\mathbf{x} = \frac{k}{y}$

O.	ıes	ŧi.	<u> </u>	16	
ωı	Jes	ST I	OI	15	_

- 1. The shutter speed, S, of a camera varies inversely as the square of the aperture setting, f. When f = 8, S = 125
  - (a) Find a formula for S in terms of f.

(3)

(b) Hence, or otherwise, calculate the value of S when f = 4

S = ......

(Total 4 marks)

2. In a factory, chemical reactions are carried out in spherical containers.

The time, *T* minutes, the chemical reaction takes is directly proportional to the square of the radius, *R* cm, of the spherical container.

When R = 120, T = 32

Find the value of T when R = 150

T = .....(Total 4 marks)

	directly proportional to the square of $t$ .  80 when $t = 4$ Express $d$ in terms of $t$ .		3.
(3)	Work out the value of $d$ when $t = 7$	(b)	
d =(1)	d = 0 Work out the positive value of $t$ when $d = 45$	(c)	
t =	distance, $D$ , travelled by a particle is directly propn. When $t = 40$ , $D = 30$ Find a formula for $D$ in terms of $t$ .		4.
D =(3)	D = Calculate the value of $D$ when $t = 64$	(b)	
<b>(1)</b> gures.	Calculate the value of $t$ when $D = 12$ Give your answer correct to 3 significant figure.	(c)	
(2) (Total 6 marks)			

The time, $T$ seconds, it takes a water heater to boil some water is directly proportional to the mass of water, $m$ kg, in the water heater. When $m = 250$ , $T = 600$ (a) Find T when $m = 400$				
	The time, $T$ seconds, it takes a water heater to be proportional to the power, $P$ watts, of the water heater $P = 1400$ , $T = 360$ (b) Find the value of $T$ when $P = 900$			
T =(3) (Total 6 marks) conds.	A ball falls vertically after being dropped.  The ball falls a distance <i>d</i> metres in a time of <i>t</i> se <i>d</i> is directly proportional to the square of <i>t</i> .  The ball falls 20 metres in a time of 2 seconds.	<b>S</b> .		
d =(3)	<ul> <li>(a) Find a formula for d in terms of t.</li> <li>c) Calculate the distance the ball falls in 3 seconds.</li> </ul>	(b)		
m <b>(1)</b>	Calculate the time the ball takes to fall 605 m.	c)		
seconds (3) (Total 7 marks)				

	is 150 newtons, the extension is 6 cm. Find a formula for $T$ in terms of $x$ .	tens (a)
(3)	$T = \dots$ Calculate the tension, in newtons, when the extension is 15 cm.	(b)
newtons <b>(1)</b>	alculate the extension, in cm, when the tension is 600 newtons.	(c)
cm (1) (Total 5 marks)	rsely proportional to $d$ . $= 50, f = 256$ $= value of f when d = 80$	Wh
(Total 3 marks)	f =	

In a spring, the tension (T newtons) is directly proportional to its extension (x cm). When the

# <u>Percentages – reverse</u>

# Things to remember:

• Work out what the multiplier would have been;

Original	x multiplier	New
amount		amount
	÷ multiplier	

_								
Q		Δ	c	tı	$\sim$	n	c	•
w	u	•	3	LI	u			-

		· muttiplier	
Quest	After	sulation reduces annual heating costs by 20%. he insulated his loft, Curtley's annual heating cost was £520. out Curtley's annual heating cost would have been, if he had not insulated his loft.	
2.	In a sa	£(Total 3 mark ale, normal prices are reduced by 20%. SALE - 20% OFF	 s)
	The sa	w bought a saddle for his horse in the sale. ale price of the saddle was £220. late the normal price of the saddle.	
		£(Total 3 mark	 S)
3.	This is Bill sa	s weekly pay this year is £240 s 20% more than her weekly pay last year. ys 'This means Hajra's weekly pay last year was £192'. wrong, Explain why.	
	(b)	Work out Hajra's weekly pay last year.	 1)
		£	

(Total 3 marks)

	(a) The price of a rail season ticket from Cambri Work out the price before this increase.	dge to London increased by £121.60
	(b) After the increase, the price of a rail season £2828.80 Work out the price before this increase.	£(2) ticket from Brighton to London was
		£(3
5.	In a sale, normal prices are reduced by 25%. The sale price of a saw is £12.75 Calculate the normal price of the saw.	(Total 5 marks)
		£(Total 3 marks
6.	In a sale, normal prices are reduced by 12%. The sale price of a DVD player is £242. Work out the normal price of the DVD player.	
		£(Total 3 marks
7.	A garage sells cars. It offers a discount of 20% off the normal price for control Dave pays £5200 cash for a car. Calculate the normal price of the car.	ash.
		£(Total 3 marks

The price of all rail season tickets to London increased by 4%.

# **Useful websites:**

# www.mathswatchvle.com

(Video explanations and questions)

Centre ID: twgash

Username: firstname

Password: lastname

# www.methodmaths.com

(Past papers online that get instantly marked)

Centre ID: wga

Username: firstname

Password: lastname

# www.hegartymaths.com

(Online tutorials and quizzes)

Login: first name and last name are case sensitive

# www.bbc.co.uk/schools/gcsebitesize /maths

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