# Trapezium Rule

A river, running between parallel banks, is 20 m wide. The depth, $y$ metres, of the river measured at a point $x$ metres from one bank is given by the formula

$$y = \frac{1}{10}x\sqrt{(20 - x)}, \qquad 0 \le x \le 20.$$

(*a*) Complete the table below, giving values of $y$ to 3 decimal places.

| $x$ | 0 | 4 | 8 | 12 | 16 | 20 |
|---|---|---|---|---|---|---|
| $y$ | 0 | | 2.771 | | | 0 |

**(2)**

(*b*) Use the trapezium rule with all the values in the table to estimate the cross-sectional area of the river.

**(4)**

Given that the cross-sectional area is constant and that the river is flowing uniformly at 2 m $s^{-1}$,

(*c*) estimate, in $m^3$, the volume of water flowing per minute, giving your answer to 3 significant figures.

**(2)**

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The speed, $v$ m $s^{-1}$, of a train at time $t$ seconds is given by

$$v = \sqrt{(1.2^t - 1)}, \qquad 0 \le t \le 30.$$

The following table shows the speed of the train at 5 second intervals.

| $t$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|
| $v$ | 0 | 1.22 | 2.28 | | 6.11 | | |

(*a*) Complete the table, giving the values of $v$ to 2 decimal places.

**(3)**

The distance, $s$ metres, travelled by the train in 30 seconds is given by

$$s = \int_0^{30} \sqrt{(1.2^t - 1)}\ \mathrm{d}t\,.$$

(*b*) Use the trapezium rule, with all the values from your table, to estimate the value of $s$.

**(3)**

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The curve $C$ has equation

$$y = x\sqrt{(x^3 + 1)}, \qquad 0 \leq x \leq 2.$$

(a) Copy and complete the table below, giving the values of $y$ to 3 decimal places at $x = 1$ and $x = 1.5$.

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 |
|---|---|---|---|---|---|
| $y$ | 0 | 0.530 | | | 6 |

**(2)**

(*b*) Use the trapezium rule, with all the $y$ values from your table, to find an approximation for the value of $\int_0^2 x\sqrt{(x^3+1)}\ dx$, giving your answer to 3 significant figures.

**(4)**

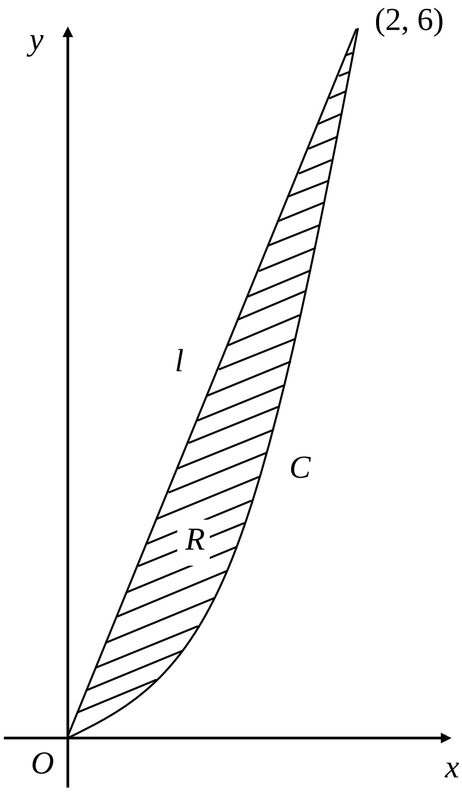


**Figure 2**

Figure 2 shows the curve $C$ with equation $y = x\sqrt{(x^3 + 1)}$, $0 \leq x \leq 2$, and the straight line segment $l$, which joins the origin and the point (2, 6). The finite region $R$ is bounded by $C$ and $l$.

(*c*) Use your answer to part (*b*) to find an approximation for the area of $R$, giving your answer to 3 significant figures.

**(4)**

($a$) Sketch the graph of $y = 3^x$, $x \in \mathbb{R}$, showing the coordinates of the point at which the graph meets the $y$-axis.

**(2)**

($b$) Copy and complete the table, giving the values of $3^x$ to 3 decimal places.

| $x$ | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
|---|---|---|---|---|---|---|
| $3^x$ | | 1.246 | 1.552 | | | 3 |

**(2)**

($c$) Use the trapezium rule, with all the values from your tables, to find an approximation for the value of $\int_0^1 3^x \, dx$.

**(4)**

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$$y = \sqrt{(5^x + 2)}$$

($a$) Copy and complete the table below, giving the values of $y$ to 3 decimal places.

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 |
|---|---|---|---|---|---|
| $y$ | | | 2.646 | 3.630 | |

**(2)**

($b$) Use the trapezium rule, with all the values of $y$ from your table, to find an approximation for the value of $\int_0^2 \sqrt{(5^x + 2)} \, dx$.

**(4)**

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$$y = \sqrt{(10x - x^2)}.$$

($a$) Copy and complete the table below, giving the values of $y$ to 2 decimal places.

| $x$ | 1 | 1.4 | 1.8 | 2.2 | 2.6 | 3 |
|---|---|---|---|---|---|---|
| $y$ | 3 | 3.47 | | | 4.39 | |

**(2)**

($b$) Use the trapezium rule, with all the values of $y$ from your table, to find an approximation for the value of $\int_1^3 \sqrt{(10x - x^2)} \, dx$.

**(4)**

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($a$) Complete the table below, giving values of $\sqrt{(2^x + 1)}$ to 3 decimal places.

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|---|---|
| $\sqrt{(2^x + 1)}$ | 1.414 | 1.554 | 1.732 | 1.957 | | | 3 |

**(2)**

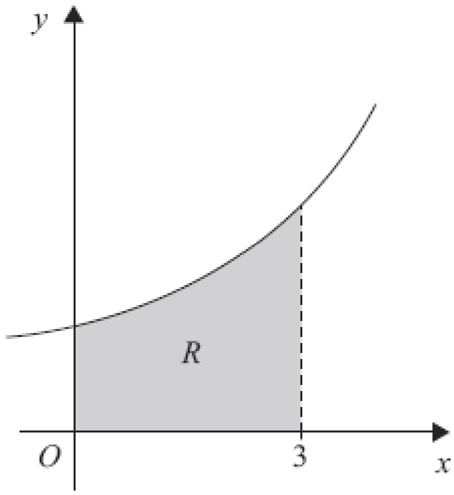


**Figure 1**

Figure 1 shows the region $R$ which is bounded by the curve with equation $y = \sqrt{(2^x + 1)}$, the $x$-axis and the lines $x = 0$ and $x = 3$

($b$) Use the trapezium rule, with all the values from your table, to find an approximation for the area of $R$.

**(4)**

($c$) By reference to the curve in Figure 1 state, giving a reason, whether your approximation in part ($b$) is an overestimate or an underestimate for the area of $R$.

**(2)**

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$$y = 3^x + 2x.$$

($a$) Complete the table below, giving the values of $y$ to 2 decimal places.

| $x$ | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
|---|---|---|---|---|---|---|
| $y$ | 1 | 1.65 | 5 | | | |

**(2)**

($b$) Use the trapezium rule, with all the values of $y$ from your table, to find an approximate value for $\int_0^1 (3^x + 2x) \, dx$.

**(4)**

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$$y = \frac{5}{3x^2 - 2}$$

($a$) Copy and complete the table below, giving the values of $y$ to 2 decimal places.

| $x$ | 2 | 2.25 | 2.5 | 2.75 | 3 |
|---|---|---|---|---|---|
| $y$ | 0.5 | 0.38 | | | 0.2 |

**(2)**

($b$) Use the trapezium rule, with all the values of $y$ from your table, to find an approximate value for $\int_2^3 \frac{5}{3x^2 - 2}\,dx$.

**(4)**

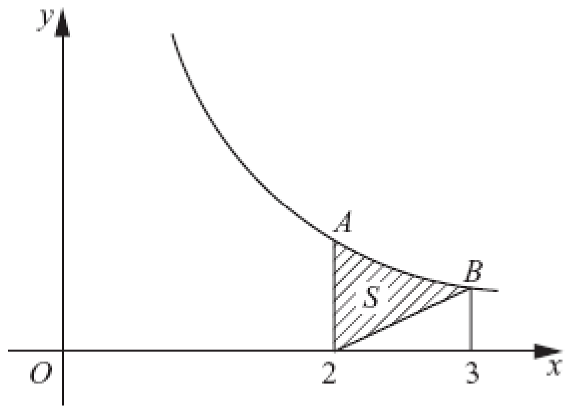


**Figure 2**

Figure 2 shows a sketch of part of the curve with equation $y = \frac{5}{3x^2 - 2}$, $x > 1$.

At the points $A$ and $B$ on the curve, $x = 2$ and $x = 3$ respectively.
The region $S$ is bounded by the curve, the straight line through $B$ and (2, 0), and the line through $A$ parallel to the $y$-axis. The region $S$ is shown shaded in Figure 2.

($c$) Use your answer to part ($b$) to find an approximate value for the area of $S$.

**(3)**