Tangents/Normals

The curve C has equation $y = 4x^2 + \frac{5 - x}{x}$, $x \neq 0$. The point P on C has x-coordinate 1.

(*a*) Show that the value of $\frac{dy}{dx}$ at P is 3.

(5)

(*b*) Find an equation of the tangent to C at P.

(3)

This tangent meets the x-axis at the point $(k, 0)$.

(*c*) Find the value of k.

(2)

The gradient of the curve C is given by

$$\frac{dy}{dx} = (3x - 1)^2.$$

The point $P(1, 4)$ lies on C.

(*a*) Find an equation of the normal to C at P.

(4)

(*b*) Find an equation for the curve C in the form $y = f(x)$.

(5)

(*c*) Using $\frac{dy}{dx} = (3x - 1)^2$, show that there is no point on C at which the tangent is parallel to the line $y = 1 - 2x$.

(2)

The curve C has equation $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$.

The point P has coordinates $(3, 0)$.

(*a*) Show that P lies on C.

(1)

(*b*) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(5)

Another point Q also lies on C. The tangent to C at Q is parallel to the tangent to C at P.

(*c*) Find the coordinates of Q.

(5)

The curve C with equation $y = \mathrm{f}(x)$, $x \neq 0$, passes through the point $(3, 7\frac{1}{2})$.

Given that $\mathrm{f}'(x) = 2x + \dfrac{3}{x^2}$,

(a) find $\mathrm{f}(x)$.

(5)

(b) Verify that $\mathrm{f}(-2) = 5$.

(1)

(c) Find an equation for the tangent to C at the point $(-2, 5)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Figure 2

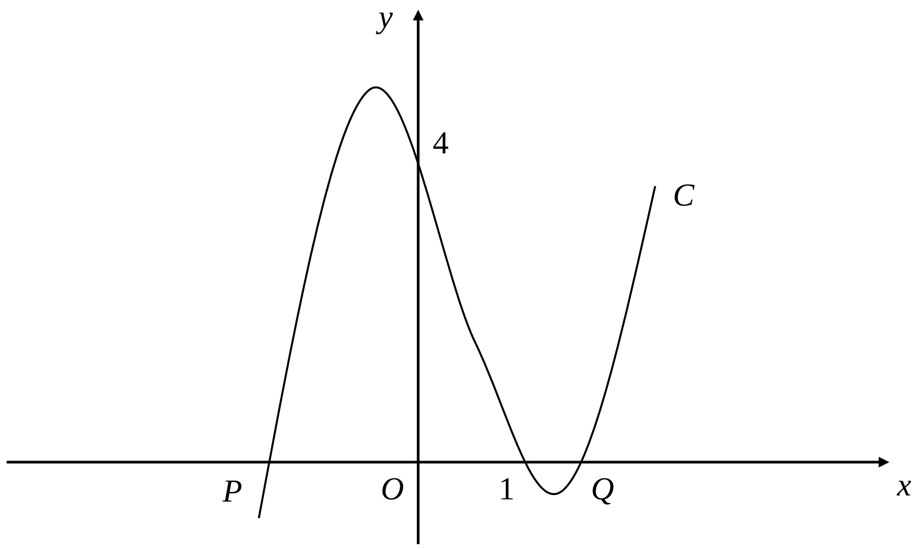

Figure 2 shows part of the curve C with equation

$$y = (x - 1)(x^2 - 4).$$

The curve cuts the x-axis at the points P, $(1, 0)$ and Q, as shown in Figure 2.

(a) Write down the x-coordinate of P and the x-coordinate of Q.

(2)

(b) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 2x - 4$.

(3)

(c) Show that $y = x + 7$ is an equation of the tangent to C at the point $(-1, 6)$.

(2)

The tangent to C at the point R is parallel to the tangent at the point $(-1, 6)$.

(d) Find the exact coordinates of R.

(5)

The curve C has equation $y = \mathrm{f}(x)$, $x \neq 0$, and the point $P(2, 1)$ lies on C. Given that

$$f'(x) = 3x^2 - 6 - \frac{8}{x^2},$$

(*a*) find f(*x*).

(5)

(*b*) Find an equation for the tangent to *C* at the point *P*, giving your answer in the form $y = mx + c$, where *m* and *c* are integers.

(4)

The curve *C* has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2, \quad x > 0$.

(*a*) Find an expression for $\frac{dy}{dx}$.

(3)

(*b*) Show that the point *P*(4, 8) lies on *C*.

(1)

(*c*) Show that an equation of the normal to *C* at the point *P* is

$$3y = x + 20.$$

(4)

The normal to *C* at *P* cuts the *x*-axis at the point *Q*.

(*d*) Find the length *PQ*, giving your answer in a simplified surd form.

(3)

The curve *C* has equation $y = x^2(x - 6) + \frac{4}{x}, \quad x > 0$.

The points *P* and *Q* lie on *C* and have *x*-coordinates 1 and 2 respectively.

(*a*) Show that the length of *PQ* is $\sqrt{170}$.

(4)

(*b*) Show that the tangents to *C* at *P* and *Q* are parallel.

(5)

(*c*) Find an equation for the normal to *C* at *P*, giving your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

(4)

The curve *C* has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2, \quad x > 0$.

(*a*) Find an expression for $\frac{dy}{dx}$.

(3)

(*b*) Show that the point $P(4, 8)$ lies on C.

(1)

(*c*) Show that an equation of the normal to C at the point P is

$$3y = x + 20.$$

(4)

The normal to C at P cuts the x-axis at the point Q.

(*d*) Find the length PQ, giving your answer in a simplified surd form.

(3)

The curve C has equation $y = f(x)$, $x > 0$, and $f'(x) = 4x - 6\sqrt{x} + \frac{8}{x^2}$.

Given that the point $P(4, 1)$ lies on C,

(*a*) find $f(x)$ and simplify your answer.

(6)

(*b*) Find an equation of the normal to C at the point $P(4, 1)$.

(4)

The curve C has equation $y = kx^3 - x^2 + x - 5$, where k is a constant.

(*a*) Find $\frac{dy}{dx}$.

(2)

The point A with x-coordinate $-\frac{1}{2}$ lies on C. The tangent to C at A is parallel to the line with equation $2y - 7x + 1 = 0$.

Find

(*b*) the value of k,

(4)

(*c*) the value of the y-coordinate of A.

(2)

The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \quad x > 0.$$

The point P on C has x-coordinate equal to 2.

(*a*) Show that the equation of the tangent to *C* at the point *P* is $y = 1 - 2x$.

(6)

(*b*) Find an equation of the normal to *C* at the point *P*.

(3)

The tangent at *P* meets the *x*-axis at *A* and the normal at *P* meets the *x*-axis at *B*.

(*c*) Find the area of the triangle *APB*.

(4)

The curve *C* has equation

$$y = x^3 - 2x^2 - x + 9, \quad x > 0.$$

The point *P* has coordinates (2, 7).

(*a*) Show that *P* lies on *C*.

(1)

(*b*) Find the equation of the tangent to *C* at *P*, giving your answer in the form $y = mx + c$, where *m* and *c* are constants.

(5)

The point *Q* also lies on *C*.

Given that the tangent to *C* at *Q* is perpendicular to the tangent to *C* at *P*,

(*c*) show that the *x*-coordinate of *Q* is $\frac{1}{3}(2 + \sqrt{6})$.

(5)

The curve *C* has equation

$$y = \frac{(x+3)(x-8)}{x}, \quad x > 0.$$

(*a*) Find $\frac{dy}{dx}$ in its simplest form.

(4)

(*b*) Find an equation of the tangent to *C* at the point where $x = 2$.

(4)

The curve *C* has equation $y = f(x)$, $x > 0$, where

$$\frac{dy}{dx} = 3x - \frac{5}{\sqrt{x}} - 2.$$

Given that the point *P* (4, 5) lies on *C*, find (*a*) f(x),

(5)

(*b*) an equation of the tangent to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The curve C has equation

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \qquad x > 0.$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(4)

(*b*) Show that the point $P(4, –8)$ lies on C.

(2)

(*c*) Find an equation of the normal to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

The curve C has equation

$$y = (x + 1)(x + 3)^2.$$

(*a*) Sketch C, showing the coordinates of the points at which C meets the axes.

(4)

(*b*) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 14x + 15$.

(3)

The point A, with x-coordinate –5, lies on C.

(*c*) Find the equation of the tangent to C at A, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Another point B also lies on C. The tangents to C at A and B are parallel.

(*d*) Find the x-coordinate of B.

(3)