CORE I REVISION

Indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{1}{m}} = \sqrt[m]{a}$$

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

$$(a^m)^n = a^{mn}$$

$$a^0 = 1$$

Evaluate:

a $\left(\frac{8}{27}\right)^{\frac{2}{3}}$

b $\left(\frac{225}{289}\right)^{\frac{3}{2}}$

Simplify:

a $9x^3 \div 3x^{-3}$

b $(4^{\frac{3}{2}})^{\frac{1}{3}}$

c $3x^{-2} \times 2x^4$

d $3x^{\frac{1}{3}} \div 6x^{\frac{2}{3}}$

Surds

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

The rules to rationalise surds are:

- Fractions in the form $\frac{1}{\sqrt{a}}$, multiply the top and bottom by $\sqrt{a}$.
- Fractions in the form $\frac{1}{a + \sqrt{b}}$, multiply the top and bottom by $a - \sqrt{b}$.
- Fractions in the form $\frac{1}{a - \sqrt{b}}$, multiply the top and bottom by $a + \sqrt{b}$.

Simplify:

a $\dfrac{3}{\sqrt{63}}$

b $\sqrt{20} + 2\sqrt{45} - \sqrt{80}$

Rationalise: $\frac{3}{\sqrt{3} - 2}$

(a) Express each of the following in the form $k\sqrt{5}$.

(i) $\sqrt{45}$

(ii) $\dfrac{20}{\sqrt{5}}$

(b) Hence write $\sqrt{45} + \dfrac{20}{\sqrt{5}}$ in the form $n\sqrt{5}$, where n is an integer.

Express each of the following in the form $p + q\sqrt{3}$.

(a) $(2+\sqrt{3})(5-2\sqrt{3})$

(b) $\dfrac{26}{4-\sqrt{3}}$

algebraic expressions

- Manipulate polynomials
- Expand brackets and collect like terms
- Factorise expressions

Rewrite the following expressions without using brackets:

a) $(s+1)(s+5)(s-3)$ b) $(x+y)^3$

Factorise

$$x^2 + 11x + 18$$

$$6x^2 + 31x + 35$$

$$-3x^2 + 2x + 5$$

Simplify

$$\frac{3}{x+2} \div \frac{x}{2x+4}$$

$$\frac{2}{2x+1} - \frac{3}{3x+2}$$

Express $\frac{1}{u} + \frac{1}{v}$ as a single fraction

Make N the subject of the formula $L = \dfrac{\mu N^2 A}{\ell}$

Equations and inequalities

1 You can solve linear simultaneous equations by elimination or substitution.

2 You can use the substitution method to solve simultaneous equations, where one equation is linear and the other is quadratic. You usually start by finding an expression for x or y from the linear equation.

3 When you multiply or divide an inequality by a negative number, you need to change the inequality sign to its opposite.

4 To solve a quadratic inequality you

- solve the corresponding quadratic equation, then
- sketch the graph of the quadratic function, then
- use your sketch to find the required set of values.

Solve the equations:

$$2x - y = 1$$
$$4x + 2y = -30$$

Show that the elimination of x from the simultaneous equations

$$x - 2y = 1$$
$$3xy - y^2 = 8$$

produces the equation

$$5y^2 + 3y - 8 = 0.$$

Solve this quadratic equation and hence find the pairs (x, y) for which the simultaneous equations are satisfied.

a Given that $3^x = 9^{y-1}$, show that $x = 2y - 2$.

b Solve the simultaneous equations:

$$x = 2y - 2$$
$$x^2 = y^2 + 7$$

a Solve the inequality $3x - 8 > x + 13$.

b Solve the inequality $x^2 - 5x - 14 > 0$.

Find the set of values of x for which:

a $6x - 7 < 2x + 3$

b $2x^2 - 11x + 5 < 0$

c both $6x - 7 < 2x + 3$ and $2x^2 - 11x + 5 < 0$.

The specification for a rectangular car park states that the length x m is to be 5 m more than the breadth. The perimeter of the car park is to be greater than 32 m.

a Form a linear inequality in x.

The area of the car park is to be less than 104 m^2.

b Form a quadratic inequality in x.

c By solving your inequalities, determine the set of possible values of x.

Quadratic functions

The general form of a quadratic equation is
$y = ax^2 + bx + c$ where a, b, c are constants and $a \neq 0$.

Quadratic equations can be solved by:

- Factorisation.
- Completing the square:

$$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

- Using the formula

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

To sketch a quadratic graph:

- Decide on the shape:

  $a > 0$ ∪

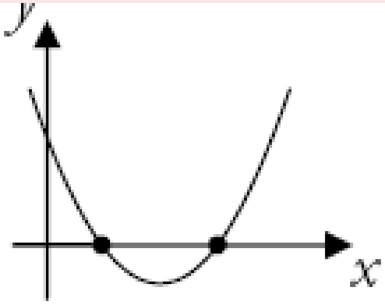
  $a < 0$ ∩

- Work out the x-axis and y-axis crossing points.
- Check the general shape by considering the discriminant $b^2 - 4ac$.

The Discriminant

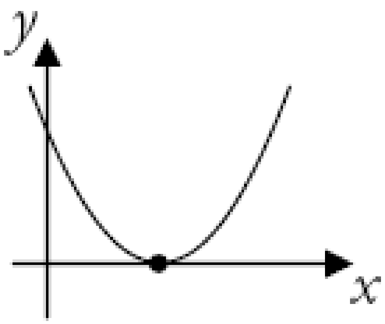
Given ax^2+bx+c, we call b^2-4ac the discriminant.

If $b^2-4ac>0$, the roots are real and unequal (distinct).



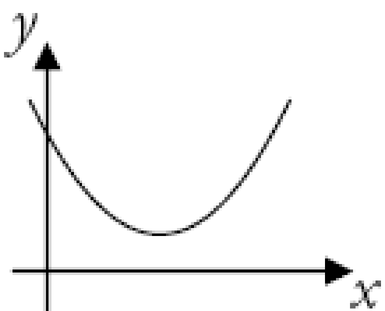
two roots

If $b^2-4ac=0$, the roots are real and equal (i.e. a repeated root).



one root

If $b^2-4ac<0$, the roots are not real; they do not exist.



no real roots

Given that for all values of x:

$$3x^2 + 12x + 5 = p(x + q)^2 + r$$

a Find the values of p, q and r.

b Solve the equation $3x^2 + 12x + 5 = 0$.

Solve the following equations by:

i Completing the square.

ii Using the formula.

a $x^2 + 5x + 2 = 0$

b $x^2 - 4x - 3 = 0$

Sketch graphs of the following equations:

a $y = x^2 + 5x + 4$

b $y = 2x^2 + x - 3$

Find the values of k for which $kx^2 + 8x + 5 = 0$ has real roots.

Coordinate Geometry

Mid-Point Between Two Points

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Length Between Two Points

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Definition Of Gradient

$$\text{Gradient} = \frac{\text{Difference in } y}{\text{Difference in } x} = \frac{y_2-y_1}{x_2-x_1}$$

Perpendicular Lines

$$m_1 \times m_2 = -1$$

The equation of a straight line with gradient m and intercept c on the y-axis is

$$y = mx + c\,.$$

The equation of a straight line with gradient m, passing through the point (x_1, y_1), is

$$y - y_1 = m(x - x_1)\,.$$

The equation of a straight line passing through the two points (x_1, y_1) and (x_2, y_2) is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}\,.$$

Transformations of graphs

Shifts

For c>0,

to obtain the graph of:

f(x)+c	*shift the graph of f(x)*	**upward** c units
f(x)-c	*shift the graph of f(x)*	**downward** c units
f(x+c)	*shift the graph of f(x)*	**left** c units
f(x-c)	*shift the graph of f(x)*	**right** c units

Reflections

To obtain the graph of:

-f(x)	*reflect the graph of f(x)*	**about the x-axis**
f(-x)	*reflect the graph of f(x)*	**about the y-axis**

Stretches and compressions

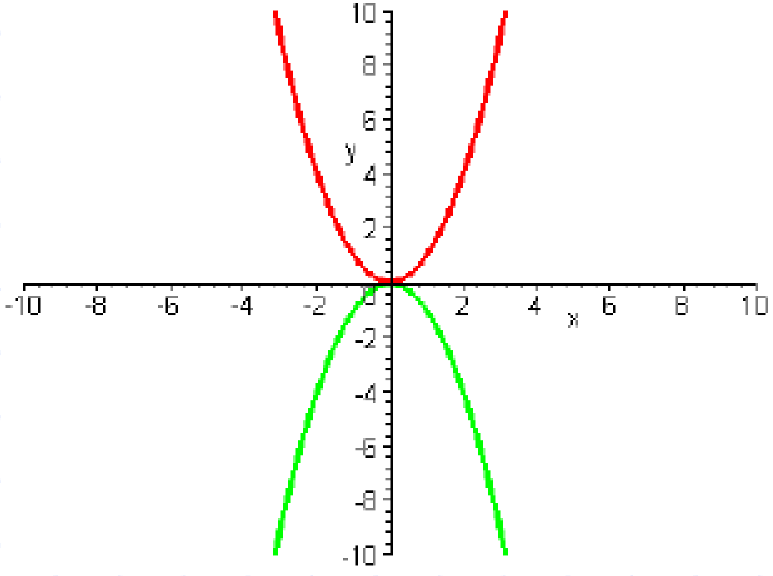
For c>1,

to obtain the graph of:

cf(x)	***stretch*** *the graph of f(x)*	**vertically** by a factor of c
(1/c)f(x)	***compress*** *the graph of f(x)*	**vertically** by a factor of c
f(cx)	***compress*** *the graph of f(x)*	**horizontally** by a factor of c
f(x/c)	***stretch*** *the graph of f(x)*	**horizontally** by a factor of c

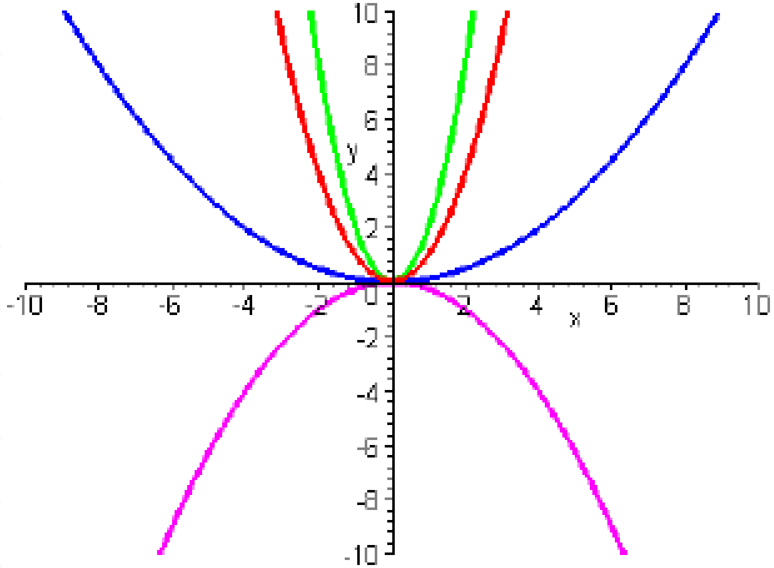
Transformations: Quadratic Equations

- In the standard graph of a quadratic equation, $y = ax^2$, the parabola opens upward when a is positive.
- When a is negative it opens downward and the graph is reflected (flipped upside down) about the x-axis: $y = - x^2$.



Transformations: Quadratic Equations

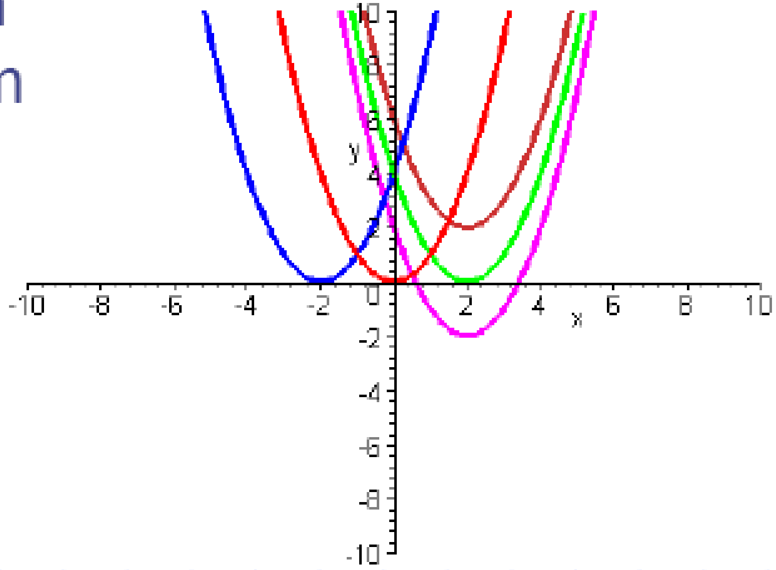
- In the standard graph of a quadratic equation $y = ax^2$, the opening of the parabola
  - narrows when the value of a increases: $y = x^2$, $y = 2x^2$
  - and widens as a decreases: $y = 1/8\ x^2$
- The graph is reflected about the x-axis when a is negative: $y = -\frac{1}{4}x^2$



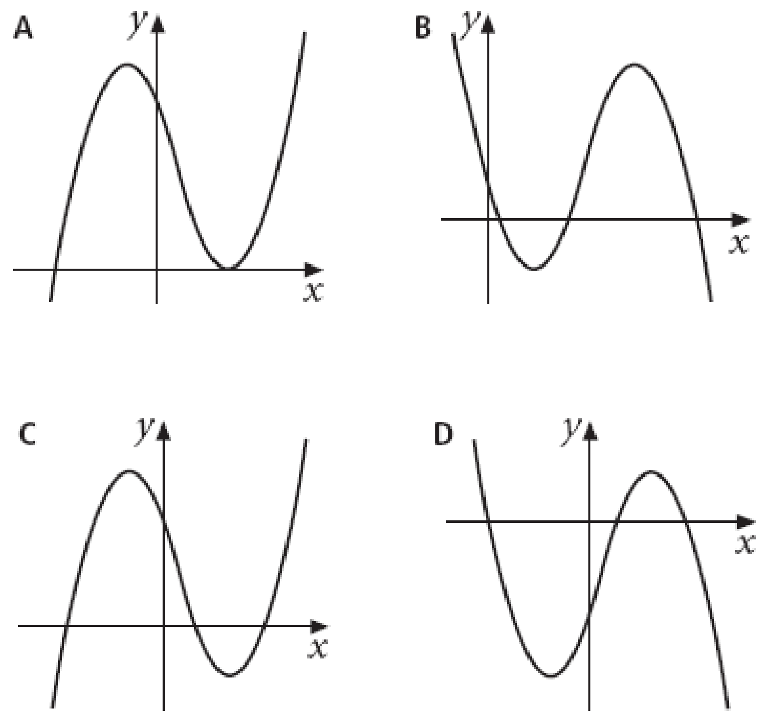
Transformations: Quadratic Equations

Displaying the quadratic equation in the "complete the square" form of $y = a(x - b)^2 + c$ makes it easy to recognize:

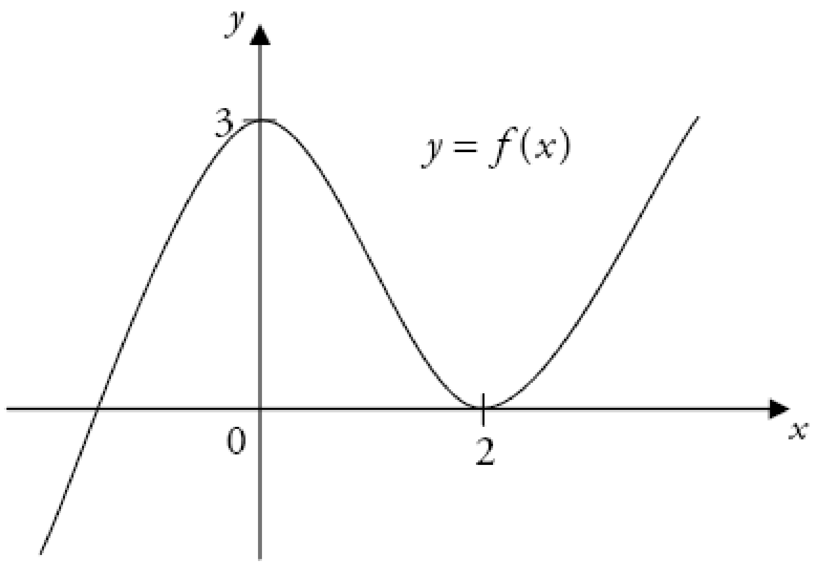
- horizontal shifts as b is changed:
  - $y = (x - 2)^2$ moves the graph of $y = x^2$ two spaces to the right,
  - and $y = (x + 2)^2$ moves it two spaces to the left.
- and vertical shifts as c is changed:
  - $y = (x - 2)^2 + 2$ shifts the graph right two spaces and up two spaces,
  - and $y = (x - 2)^2 - 2$ shifts the graph right two spaces and down two spaces.



Decide which of the graphs below is a sketch of $y = -x^3 + 3x^2 + 18x - 40$.



The diagram shows part of the graph of $y = f(x)$.



On separate diagrams, sketch the graphs of:

a) $y = -f(x)$

b) $y = f(x+4)$

Sketch the graph of $y = (x + 2)^3 + 5$.

An arithmetic progression, or AP, is a sequence where each new term after the first is obtained by adding a constant d, called the *common difference*, to the preceding term. If the first term of the sequence is a then the arithmetic progression is

$$a, \quad a+d, \quad a+2d, \quad a+3d, \quad \ldots$$

where the n-th term is $a+(n-1)d$.

The sum of the terms of an arithmetic progression gives an arithmetic series. If the starting value is a and the common difference is d then the sum of the first n terms is

$$S_n = \tfrac{1}{2}n(2a+(n-1)d)\,.$$

If we know the value of the last term ℓ instead of the common difference d then we can write the sum as

$$S_n = \tfrac{1}{2}n(a+\ell)\,.$$

Evaluate $\sum_{r=1}^{21}(4+7r)$

A salesman is paid commission of £10 per week for each life insurance policy which he has sold.

Each week he sells one new policy so that he is paid £10 commission in the first week, £20 in the second week, £30 in the third week and so on.

Find his total commission in the first year of 52 weeks.

A sequence $a_1, a_2, a_3, \ldots$ is defined by

$a_1 = 3$,

$a_{n+1} = 3a_n - 5, \quad n \geq 1$.

(*a*) Find the value a_2 and the value of a_3.

(*b*) Calculate the value of $\sum_{r=1}^{5} a_r$

differentiation

$$\text{if } y = x^n \quad \text{then} \quad \frac{dy}{dx} = nx^{n-1}$$

If $f(x) = x^4 + 2x^3$, find $f'(x)$.

Find the gradient of the graph of $y = x^3 - 6x$ at the point where $x = -1$.

Integration

$$\int x^n \mathrm{d}x = \frac{x^{n+1}}{n+1} + c$$

Find $\int (x+1)(x-2)(x-4)\,\mathrm{d}x$.

(a) Find the equation of the normal to the graph of $y = 5x - x^2$ at the point $(3, 6)$.

(b) This normal intersects the graph again at the point A. Find the coordinates of A.

The graph of $y = 4x^3 + ax + b$ goes through the point $P(-1, 1)$.
The gradient of the graph at P is 14.
Find the values of a and b.

Find the equations of the tangent and the normal to $y = x^3 - 10x + 1$ at the point P where $x = 2$.

A curve has equation $y = x^3 + 3x^2 - 7x - 1$.

(a) Find $\frac{dy}{dx}$ in terms of x.

Points P and Q lie on the curve. The gradient at both P and Q is 2.
The x-coordinate of P is 1.

(b) Find the x-coordinate of Q.

(c) Find an equation for the tangent at P, giving your answer in the form $y = mx + c$, where m and c are constants.

This tangent intersects the coordinate axes at points R and S.

(d) Find the length of RS, giving your answer as a surd.