Co-ordinate Geometry

The straight line l_1 has equation $y = 3x - 6$.

The straight line l_2 is perpendicular to l_1 and passes through the point (6, 2).

(a) Find an equation for l_2 in the form $y = mx + c$, where m and c are constants.

(3)

The lines l_1 and l_2 intersect at the point C.

(b) Use algebra to find the coordinates of C.

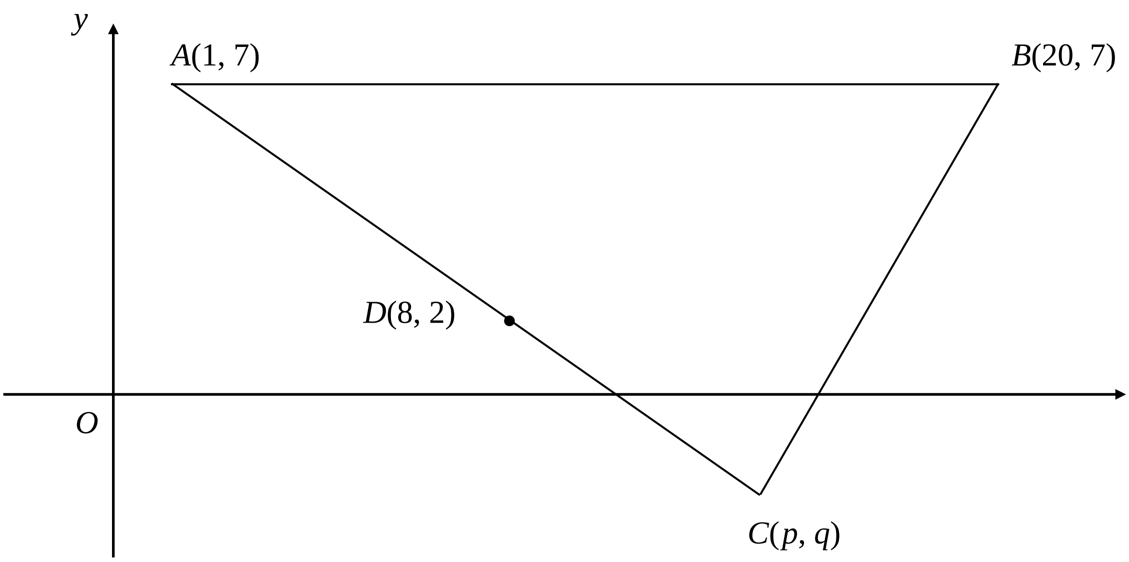
(3)

The lines l_1 and l_2 cross the x-axis at the points A and B respectively.

(c) Calculate the exact area of triangle ABC.

(4)

Figure 2



The points $A(1, 7)$, $B(20, 7)$ and $C(p, q)$ form the vertices of a triangle ABC, as shown in Figure 2. The point $D(8, 2)$ is the mid-point of AC.

(a) Find the value of p and the value of q.

(2)

The line l, which passes through D and is perpendicular to AC, intersects AB at E.

(b) Find an equation for l, in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

(c) Find the exact x-coordinate of E.

(2)

Given that

$$f(x) = x^2 - 6x + 18, \quad x \geq 0,$$

(*a*) express f(*x*) in the form $(x - a)^2 + b$, where *a* and *b* are integers.

(3)

The curve *C* with equation $y = f(x)$, $x \geq 0$, meets the *y*-axis at *P* and has a minimum point at *Q*.

(*b*) Sketch the graph of *C*, showing the coordinates of *P* and *Q*.

(4)

The line $y = 41$ meets *C* at the point *R*.

(*c*) Find the *x*-coordinate of *R*, giving your answer in the form $p + q\sqrt{2}$, where *p* and *q* are integers.

(5)

The line l_1 passes through the point (9, –4) and has gradient $\frac{1}{3}$.

(*a*) Find an equation for l_1 in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

(3)

The line l_2 passes through the origin *O* and has gradient –2. The lines l_1 and l_2 intersect at the point *P*.

(*b*) Calculate the coordinates of *P*.

(4)

Given that l_1 crosses the *y*-axis at the point *C*,

(*c*) calculate the exact area of $\triangle OCP$.

(3)

The line *L* has equation $y = 5 - 2x$.

(*a*) Show that the point *P* (3, –1) lies on *L*.

(1)

(*b*) Find an equation of the line perpendicular to *L*, which passes through *P*. Give your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

(4)

The line l_1 passes through the points $P(-1, 2)$ and $Q(11, 8)$.

(*a*) Find an equation for l_1 in the form $y = mx + c$, where m and c are constants.

(4)

The line l_2 passes through the point $R(10, 0)$ and is perpendicular to l_1. The lines l_1 and l_2 intersect at the point S.

(*b*) Calculate the coordinates of S.

(5)

(*c*) Show that the length of RS is $3\sqrt{5}$.

(2)

(*d*) Hence, or otherwise, find the exact area of triangle PQR.

(4)

The line l_1 has equation $y = 3x + 2$ and the line l_2 has equation $3x + 2y - 8 = 0$.

(*a*) Find the gradient of the line l_2.

(2)

The point of intersection of l_1 and l_2 is P.

(*b*) Find the coordinates of P.

(3)

The lines l_1 and l_2 cross the line $y = 1$ at the points A and B respectively.

(*c*) Find the area of triangle ABP.

(4)

The point $A(-6, 4)$ and the point $B(8, -3)$ lie on the line L.

(*a*) Find an equation for L in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

(*b*) Find the distance AB, giving your answer in the form $k\sqrt{5}$, where k is an integer.

(3)

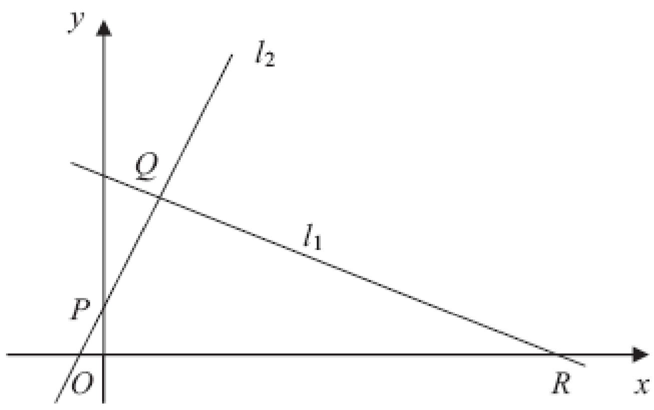


Figure 2

The points Q (1, 3) and R (7, 0) lie on the line l_1, as shown in Figure 2.

The length of QR is $a\sqrt{5}$.

(*a*) Find the value of a.

(3)

The line l_2 is perpendicular to l_1, passes through Q and crosses the y-axis at the point P, as shown in Figure 2. Find

(*b*) an equation for l_2,

(5)

(*c*) the coordinates of P,

(1)

(*d*) the area of ΔPQR.

(4)

The line l_1 passes through the point $A(2, 5)$ and has gradient $-\frac{1}{2}$.

(*a*) Find an equation of l_1, giving your answer in the form $y = mx + c$.

(3)

The point B has coordinates (–2, 7).

(*b*) Show that B lies on l_1.

(1)

(*c*) Find the length of AB, giving your answer in the form $k\sqrt{5}$, where k is an integer.

(3)

The point C lies on l_1 and has x-coordinate equal to p.

The length of AC is 5 units.

(*d*) Show that p satisfies

$$p^2 - 4p - 16 = 0.$$

(4)

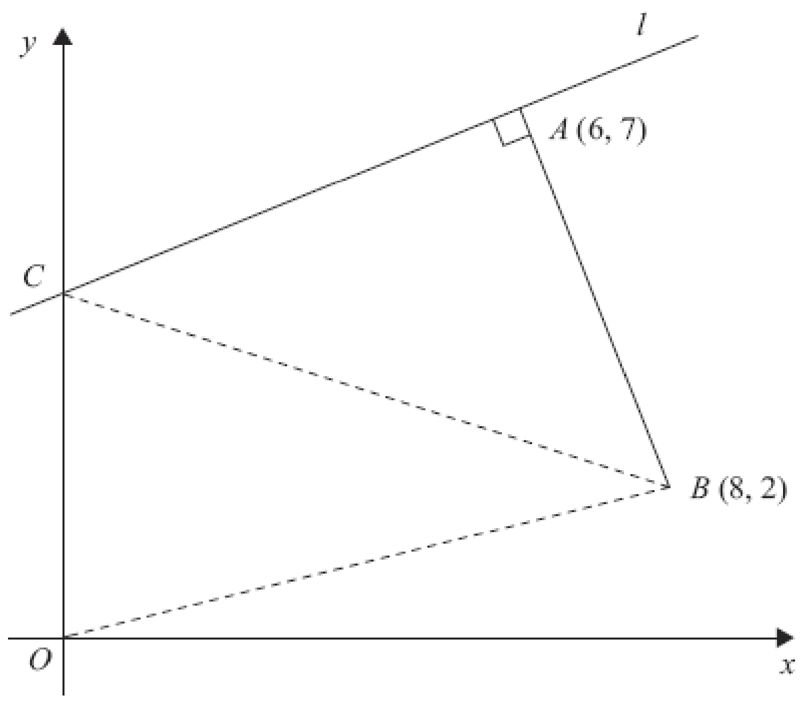


Figure 1

The points A and B have coordinates (6, 7) and (8, 2) respectively.

The line l passes through the point A and is perpendicular to the line AB, as shown in Figure 1.

(a) Find an equation for l in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Given that l intersects the y-axis at the point C, find

(b) the coordinates of C,

(2)

(c) the area of ΔOCB, where O is the origin.

(2)

The line l_1 has equation $3x + 5y - 2 = 0$.

(a) Find the gradient of l_1.

(2)

The line l_2 is perpendicular to l_1 and passes through the point (3, 1).

(b) Find the equation of l_2 in the form $y = mx + c$, where m and c are constants.

(3)

(a) Factorise completely $x^3 - 4x$.

(3)

(b) Sketch the curve C with equation

$$y = x^3 - 4x,$$

showing the coordinates of the points at which the curve meets the axis.

(3)

The point A with x-coordinate -1 and the point B with x-coordinate 3 lie on the curve C.

(c) Find an equation of the line which passes through A and B, giving your answer in the form $y = mx + c$, where m and c are constants.

(5)

(d) Show that the length of AB is $k\sqrt{10}$, where k is a constant to be found.

(2)

(a) Find an equation of the line joining $A(7, 4)$ and $B(2, 0)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

(b) Find the length of AB, leaving your answer in surd form.

(2)

The point C has coordinates $(2, t)$, where $t > 0$, and $AC = AB$.

(c) Find the value of t.

(1)

(d) Find the area of triangle ABC.

(2)

The line L_1 has equation $2y - 3x - k = 0$, where k is a constant.

Given that the point $A(1, 4)$ lies on L_1, find

(a) the value of k,

(1)

(b) the gradient of L_1.

(2)

The line L_2 passes through A and is perpendicular to L_1.

(c) Find an equation of L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The line L_2 crosses the x-axis at the point B.

(d) Find the coordinates of B.

(2)

(e) Find the exact length of AB.

(2)

The points P and Q have coordinates $(-1, 6)$ and $(9, 0)$ respectively.

The line l is perpendicular to PQ and passes through the mid-point of PQ.

Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)