Pure Mathematics – Coordinate Geometry

1. A line l_1 has equation $5y + 4x = 3$.

   (i) Find the gradient of the line. [1]

   (ii) Find the equation of the line l_2 which is parallel to l_1 and passes through the point (1, -2). [3]

2. Describe fully the curve whose equation is $x^2 + y^2 = 4$. [2]

3. The coordinates of two points are A (-1, -3) and B (5, 7). Calculate the equation of the perpendicular bisector of AB. [4]

4. Show that the line $y = 3x - 10$ is a tangent to the circle $x^2 + y^2 = 10$. [4]

5. The line $y = 2x - 3$ meets the x-axis at the point P, and the line $3y + 4x = 8$ meets the x-axis at the point Q. The two lines intersect at the point R.

   (i) Find the coordinates of R. [4]

   (ii) Find the area of triangle PQR. [3]

6. The equation of a circle is $x^2 + y^2 - 4x + 2y = 15$

   (i) Find the coordinates of the centre C of the circle, and the radius of the circle. [3]

   (ii) Show that the point P (4, -5) lies on the circle. [1]

   (iii) Find the equation of the tangent to the circle at the point P. [4]

7. The coordinates of four points are P (-2, -1), Q (6, 3), R (9, 2) and S (1, -2).

   (i) Calculate the gradients of the lines PQ, QR, RS and SP. [4]

   (ii) What name is given to the quadrilateral PQRS? [1]

   (iii) Calculate the length SR. [2]

   (iv) Show that the equation of SR is $2y = x - 5$ and find the equation of the line L through Q perpendicular to SR. [5]

   (v) Calculate the coordinates of the point T where the line L meets SR. [3]

   (vi) Calculate the area of the quadrilateral PQRS. [3]

8. AB is the diameter of a circle. A is (1, 3) and B is (7, -1).

   (i) Find the coordinates of the centre C of the circle. [2]

   (ii) Find the radius of the circle. [2]

   (iii) Find the equation of the circle. [2]

   (iv) The line $y + 5x = 8$ cuts the circle at A and again at a second point D. Calculate the coordinates of D. [4]

   (v) Prove that the line AB is perpendicular to the line CD. [3]

Total 60 marks

Coordinate Geometry Assessment Solutions

Solutions

1. (i) $5y + 4x = 3$.

$5y = -4x + 3$

$y = -\frac{4}{5}x + \frac{3}{5}$

Gradient of line = $-\frac{4}{5}$

(ii) l_2 is parallel to l_1, so it has gradient $-\frac{4}{5}$**.**

Equation of line is $y - (-2) = -\frac{4}{5}(x - 1)$

$5(y + 2) = -4(x - 1)$

$5y + 10 = -4x + 4$

$5y + 4x + 6 = 0$

2. The curve is a circle, centre O and radius 2.

3. Gradient of AB $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{-3 - 7}{-1 - 5} = \dfrac{-10}{-6} = \dfrac{5}{3}$

Gradient of line perpendicular to AB $= -\dfrac{3}{5}$**.**

The line passes through the midpoint of AB $= \left(\dfrac{-1 + 5}{2}, \dfrac{-3 + 7}{2}\right) = (2, 2)$

Equation of line is $y - 2 = -\frac{3}{5}(x - 5)$

$5(y - 2) = -3(x - 2)$

$5y - 10 = -3x + 6$

$5y + 3x = 16$

4. Substituting $y = 3x - 10$ **into** $x^2 + y^2 = 10$

gives $x^2 + (3x - 10)^2 = 10$

$x^2 + 9x^2 - 60x + 100 = 10$

$10x^2 - 60x + 90 = 0$

$x^2 - 6x + 9 = 0$

$(x - 3)^2 = 0$

Since the equation has a repeated root, the line meets the circle just once, and so the line is a tangent to the circle.

Coordinate Geometry Assessment Solutions

5. (i) Substituting $y = 2x - 3$ **into** $3y + 4x = 8$ **:**

$$3(2x-3)+4x=8$$
$$6x-9+4x=8$$
$$10x=17$$
$$x=1.7$$

When x = 1.7, $y = 2\times 1.7 - 3 = 3.4 - 3 = 0.4$

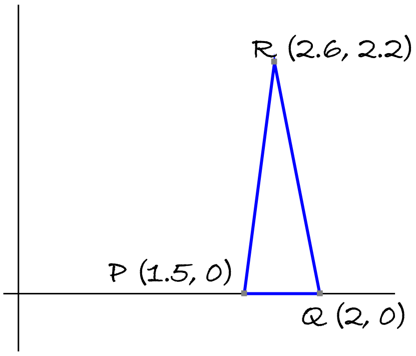
The coordinates of R are (1.7, 0.4)

(ii) P is the point on $y = 2x - 3$ **where y = 0, so P is (1.5,**

Q is the point on $3y + 4x = 8$ **where y = 0, so Q is (2,**

Area $= \frac{1}{2}\times \text{base} \times \text{height}$

$$= \tfrac{1}{2}\times 0.5 \times 0.4$$
$$= 0.1$$



6. (i) $x^2 + y^2 - 4x + 2y = 15$

$$x^2 - 4x + y^2 + 2y = 15$$
$$(x-2)^2 - 4 + (y+1)^2 - 1 = 15$$
$$(x-2)^2 + (y+1)^2 = 20$$

The centre C of the circle is (2, -1) and the radius is $\sqrt{20}$ **.**

(ii) Substituting x = 4 and y = -5: $(4-2)^2 + (-5+1)^2 = 4 + 16 = 20$
so the point (4, -5) lies on the circle.

(iii) Gradient of CP $= \dfrac{-1-(-5)}{2-4} = \dfrac{4}{-2} = -2$

Tangent at P is perpendicular to CP, so gradient of tangent $= \frac{1}{2}$**.**
Equation of tangent is $y - (-5) = \frac{1}{2}(x-4)$

$$2(y+5) = x - 4$$
$$2y + 10 = x - 4$$
$$2y = x - 14$$

7. (i) Gradient of PQ $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{-1-3}{-2-6} = \dfrac{-4}{-8} = \dfrac{1}{2}$

Gradient of QR $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{3-2}{6-9} = \dfrac{1}{-3} = -\dfrac{1}{3}$

Gradient of RS $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{2-(-2)}{9-1} = \dfrac{4}{8} = \dfrac{1}{2}$

Gradient of SP $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{-2-(-1)}{1-(-2)} = \dfrac{-1}{-3} = -\dfrac{1}{3}$

(ii) PQ is parallel to RS, and QR is parallel to SP, so the quadrilateral is a parallelogram.

Coordinate Geometry Assessment Solutions

(iii) $SR = \sqrt{(9-1)^2 + (2-(-2))^2} = \sqrt{64+16} = \sqrt{80}$

(iv) From (i), gradient of SR $= \frac{1}{2}$

Equation of SR is $y-(-2) = \frac{1}{2}(x-1)$

$$2(y+2) = x-1$$
$$2y+4 = x-1$$
$$2y = x-5$$

Line perpendicular to SR has gradient -2
Line L has gradient -2 and goes through (6, 3)
Equation of L is $y-3 = -2(x-6)$

$$y-3 = -2x+12$$
$$y+2x = 15$$

(v) Equation of L is $y = 15-2x$

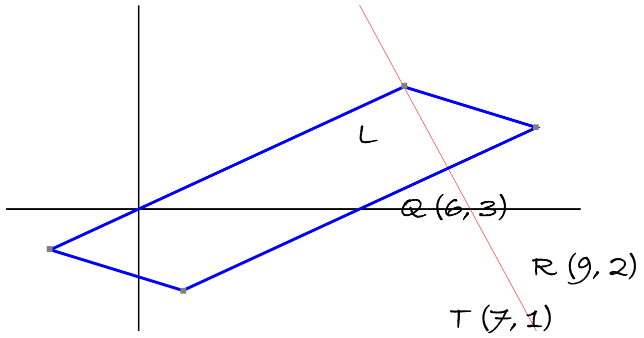
Substituting into equation of SR gives $2(15-2x) = x-5$

$$30-4x = x-5$$
$$35 = 5x$$
$$x = 7$$

When x = 7, $y = 15-2\times 7 = 1$

Coordinates of T are (7, 1)

(vi)



Length QT $= \sqrt{(7-6)^2 + (1-3)^2} = \sqrt{1+4} = \sqrt{5}$

Area of parallelogram $= SR \times QT$

$$= \sqrt{80}\sqrt{5}$$
$$= \sqrt{16}\sqrt{5}\sqrt{5}$$
$$= 4\times 5 = 20$$

8. (i) C is the midpoint of AB.

C $= \left(\frac{1+7}{2}, \frac{3+(-1)}{2}\right) = (4, 1)$

Coordinate Geometry Assessment Solutions

(ii) Radius of circle = CA $= \sqrt{(4-1)^2 + (1-3)^2} = \sqrt{9+4} = \sqrt{13}$

(iii) Equation of circle is $(x-4)^2 + (y-1)^2 = 13$

(iv) Substituting $y = -5x + 8$ **into equation of circle:**

$$(x-4)^2 + (-5x+8-1)^2 = 13$$
$$(x-4)^2 + (-5x+7)^2 = 13$$
$$x^2 - 8x + 16 + 25x^2 - 70x + 49 = 13$$
$$26x^2 - 78x + 52 = 0$$
$$x^2 - 3x + 2 = 0$$
$$(x-1)(x-2) = 0$$
$$x = 1 \text{ or } x = 2$$

x = 1 is point A, so point D is x = 2
When x = 2, $y = -5 \times 2 + 8 = -2$
The coordinates of D are (2, -2)

(v) Gradient of AB $= \dfrac{3-(-1)}{1-7} = \dfrac{4}{-6} = -\dfrac{2}{3}$

Gradient of CD $= \dfrac{1-(-2)}{4-2} = \dfrac{3}{2}$

Gradient of AB × gradient of CD $= -\dfrac{2}{3} \times \dfrac{3}{2} = -1$

so AB is perpendicular to CD.