Arithmetic Series

Ahmed plans to save £250 in the year 2001, £300 in 2002, £350 in 2003, and so on until the year 2020. His planned savings form an arithmetic sequence with common difference £50.

(a) Find the amount he plans to save in the year 2011. **(2)**

(b) Calculate his total planned savings over the 20 year period from 2001 to 2020. **(3)**

Ben also plans to save money over the same 20 year period. He saves £*A* in the year 2001 and his planned yearly savings form an arithmetic sequence with common difference £60.

Given that Ben' s total planned savings over the 20 year period are equal to Ahmed's total planned savings over the same period,

(c) calculate the value of *A*. **(4)**

An arithmetic series has first term *a* and common difference *d*.

(a) Prove that the sum of the first *n* terms of the series is

$$\frac{1}{2}n[2a + (n-1)d].$$ **(4)**

A polygon has 16 sides. The lengths of the sides of the polygon, starting with the shortest side, form an arithmetic sequence with common difference *d* cm.

The longest side of the polygon has length 6 cm and the perimeter of the polygon is 72 cm.
Find

(b) the length of the shortest side of the polygon, **(5)**

(c) the value of *d*. **(2)**

The *r*th term of an arithmetic series is $(2r - 5)$.

(*a*) Write down the first three terms of this series. **(2)**

(*b*) State the value of the common difference. **(1)**

(*c*) Show that $\sum_{r=1}^{n}(2r-5) = n(n-4)$. **(3)**

An arithmetic series has first term a and common difference d.

(a) Prove that the sum of the first n terms of the series is

$$\tfrac{1}{2}n[2a + (n - 1)d].$$

(4)

Sean repays a loan over a period of n months. His monthly repayments form an arithmetic sequence.

He repays £149 in the first month, £147 in the second month, £145 in the third month, and so on. He makes his final repayment in the nth month, where $n > 21$.

(b) Find the amount Sean repays in the 21st month.

(2)

Over the n months, he repays a total of £5000.

(c) Form an equation in n, and show that your equation may be written as

$$n^2 - 150n + 5000 = 0.$$

(3)

(d) Solve the equation in part (c).

(3)

(e) State, with a reason, which of the solutions to the equation in part (c) is **not** a sensible solution to the repayment problem.

(1)

On Alice's 11th birthday she started to receive an annual allowance. The first annual allowance was £500 and on each following birthday the allowance was increased by £200.

(a) Show that, immediately after her 12th birthday, the total of the allowances that Alice had received was £1200.

(1)

(b) Find the amount of Alice's annual allowance on her 18th birthday.

(2)

(c) Find the total of the allowances that Alice had received up to and including her 18th birthday.

(3)

When the total of the allowances that Alice had received reached £32 000 the allowance stopped.

(d) Find how old Alice was when she received her last allowance. **(7)**

An athlete prepares for a race by completing a practice run on each of 11 consecutive days. On each day after the first day he runs further than he ran on the previous day. The lengths of his 11 practice runs form an arithmetic sequence with first term a km and common difference d km.

He runs 9 km on the 11th day, and he runs a total of 77 km over the 11 day period.

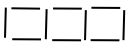
Find the value of a and the value of d.

(7)

Ann has some sticks that are all of the same length. She arranges them in squares and has made the following 3 rows of patterns:

Row 1 

Row 2 

Row 3 

She notices that 4 sticks are required to make the single square in the first row, 7 sticks to make 2 squares in the second row and in the third row she needs 10 sticks to make 3 squares.

(*a*) Find an expression, in terms of n, for the number of sticks required to make a similar arrangement of n squares in the nth row.

(3)

Ann continues to make squares following the same pattern. She makes 4 squares in the 4th row and so on until she has completed 10 rows.

(*b*) Find the total number of sticks Ann uses in making these 10 rows.

(3)

Ann started with 1750 sticks. Given that Ann continues the pattern to complete k rows but does not have sufficient sticks to complete the $(k + 1)$th row,

(*c*) show that k satisfies $(3k - 100)(k + 35) < 0$.

(4)

(*d*) Find the value of k.

(2)

A girl saves money over a period of 200 weeks. She saves 5p in Week 1, 7p in Week 2, 9p in Week 3, and so on until Week 200. Her weekly savings form an arithmetic sequence.

(*a*) Find the amount she saves in Week 200.

(3)

(*b*) Calculate her total savings over the complete 200 week period.

(3)

The first term of an arithmetic sequence is 30 and the common difference is -1.5.

(*a*) Find the value of the 25th term.

(2)

The rth term of the sequence is 0.

(*b*) Find the value of r.

(2)

The sum of the first n terms of the sequence is S_n.

(*c*) Find the largest positive value of S_n.

(3)

Sue is training for a marathon. Her training includes a run every Saturday starting with a run of 5 km on the first Saturday. Each Saturday she increases the length of her run from the previous Saturday by 2 km.

(*a*) Show that on the 4th Saturday of training she runs 11 km.

(1)

(*b*) Find an expression, in terms of n, for the length of her training run on the nth Saturday.

(2)

(*c*) Show that the total distance she runs on Saturdays in n weeks of training is $n(n + 4)$ km.

(3)

On the nth Saturday Sue runs 43 km.

(*d*) Find the value of n.

(2)

(*e*) Find the total distance, in km, Sue runs on Saturdays in n weeks of training.

(2)

The first term of an arithmetic series is a and the common difference is d.

The 18th term of the series is 25 and the 21st term of the series is $32\frac{1}{2}$.

(*a*) Use this information to write down two equations for a and d.

(2)

(*b*) Show that $a = -17.5$ and find the value of d.

(2)

The sum of the first n terms of the series is 2750.

(*c*) Show that n is given by

$$n^2 - 15n = 55 \times 40.$$

(4)

(*d*) Hence find the value of n.

(3)

A 40-year building programme for new houses began in Oldtown in the year 1951 (Year 1) and finished in 1990 (Year 40).

The numbers of houses built each year form an arithmetic sequence with first term a and common difference d.
Given that 2400 new houses were built in 1960 and 600 new houses were built in 1990, find

(a) the value of d,

(3)

(b) the value of a,

(2)

(c) the total number of houses built in Oldtown over the 40-year period.

(3)

Jill gave money to a charity over a 20-year period, from Year 1 to Year 20 inclusive. She gave £150 in Year 1, £160 in Year 2, £170 in Year 3, and son on, so that the amounts of money she gave each year formed an arithmetic sequence.

(a) Find the amount of money she gave in Year 10.

(2)

(b) Calculate the total amount of money she gave over the 20-year period.

(3)

Kevin also gave money to charity over the same 20-year period.

He gave £A in Year 1 and the amounts of money he gave each year increased, forming an arithmetic sequence with common difference £30.

The total amount of money that Kevin gave over the 20-year period was **twice** the total amount of money that Jill gave.

(c) Calculate the value of A.

(4)

A farmer has a pay scheme to keep fruit pickers working throughout the 30 day season.

He pays £a for their first day, £$(a+d)$ for their second day, £$(a+2d)$ for their third day, and so on, thus increasing the daily payment by £d for each extra day they work.

A picker who works for all 30 days will earn £40.75 on the final day.
(a) Use this information to form an equation in a and d. **(2)**

A picker who works for all 30 days will earn a total of £1005
(b) Show that $15(a+40.75)=1005$ **(2)**

(c) Hence find the value of a and the value of d. **(4)**

An arithmetic sequence has first term a and common difference d. The sum of the first 10 terms of the sequence is 162.

(a) Show that $10a + 45d = 162$.

(2)

Given also that the sixth term of the sequence is 17,

(b) write down a second equation in a and d,

(1)

(c) find the value of a and the value of d.

(4)

(a) Calculate the sum of all the even numbers from 2 to 100 inclusive,

$$2 + 4 + 6 + \ldots\ldots + 100.$$

(3)

(b) In the arithmetic series

$$k + 2k + 3k + \ldots\ldots + 100,$$

k is a positive integer and k is a factor of 100.

(i) Find, in terms of k, an expression for the number of terms in this series.

(ii) Show that the sum of this series is

$$50 + \frac{5000}{k}.$$

(4)

(c) Find, in terms of k, the 50th term of the arithmetic sequence

$$(2k + 1),\quad (4k + 4),\quad (6k + 7),\ \ldots\ ,$$

giving your answer in its simplest form.

(4)