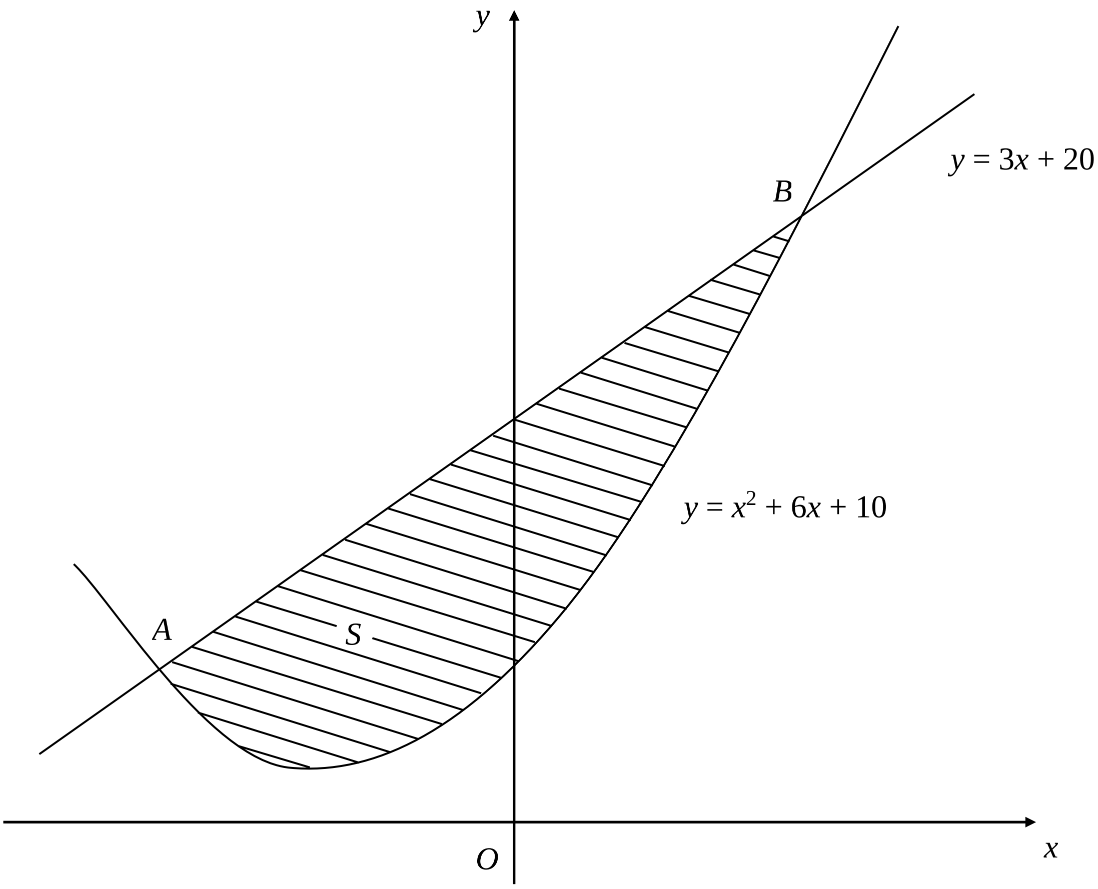
Areas/Integration

Figure 2



The line with equation $y = 3x + 20$ cuts the curve with equation $y = x^2 + 6x + 10$ at the points A and B, as shown in Figure 2.

(a) Use algebra to find the coordinates of A and the coordinates of B.

(5)

The shaded region S is bounded by the line and the curve, as shown in Figure 2.

(b) Use calculus to find the exact area of S.

(7)

Figure 1

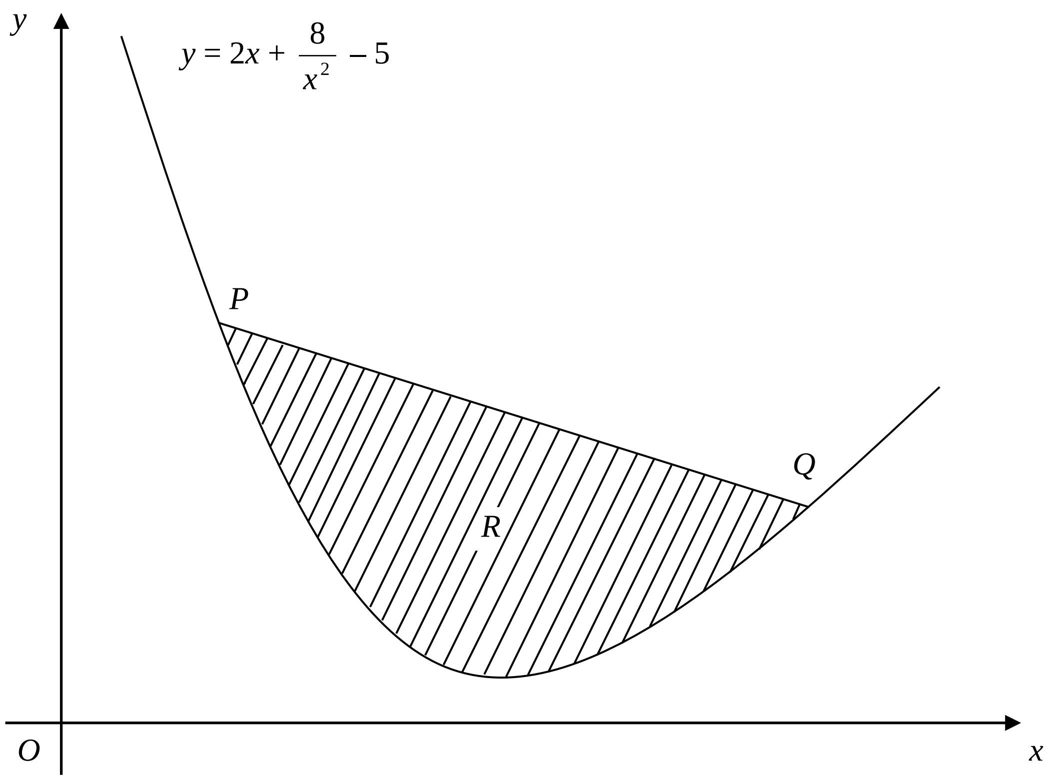


Figure 1 shows part of a curve C with equation $y = 2x + \frac{8}{x^2} - 5$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 4 respectively. The region R, shaded in Figure 1, is bounded by C and the straight line joining P and Q.

(a) Find the exact area of R.

(8)

(b) Use calculus to show that y is increasing for $x > 2$.

(4)

Figure 3

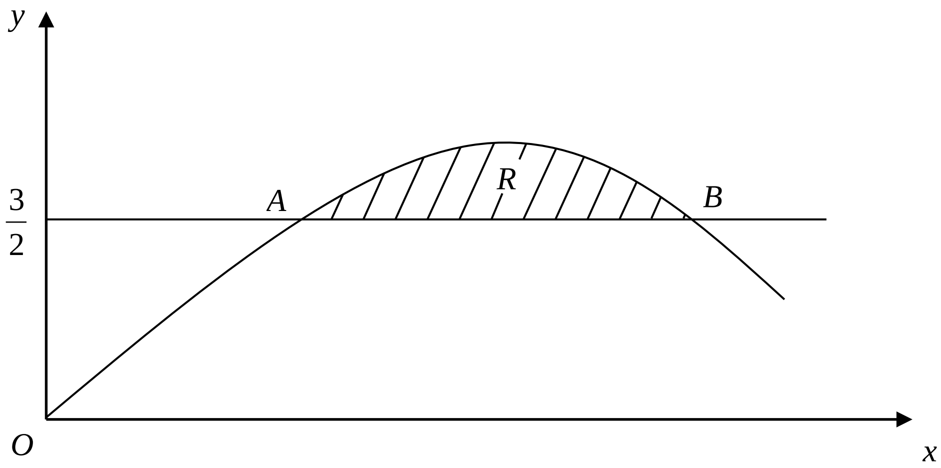


Figure 3 shows the shaded region R which is bounded by the curve $y = -2x^2 + 4x$ and the line $y = \frac{3}{2}$. The points A and B are the points of intersection of the line and the curve.

Find

(a) the x-coordinates of the points A and B,

(4)

(b) the exact area of R.

(6)

Use calculus to find the exact value of $\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2}\right) \mathrm{d}x$.

(5)

Figure 3

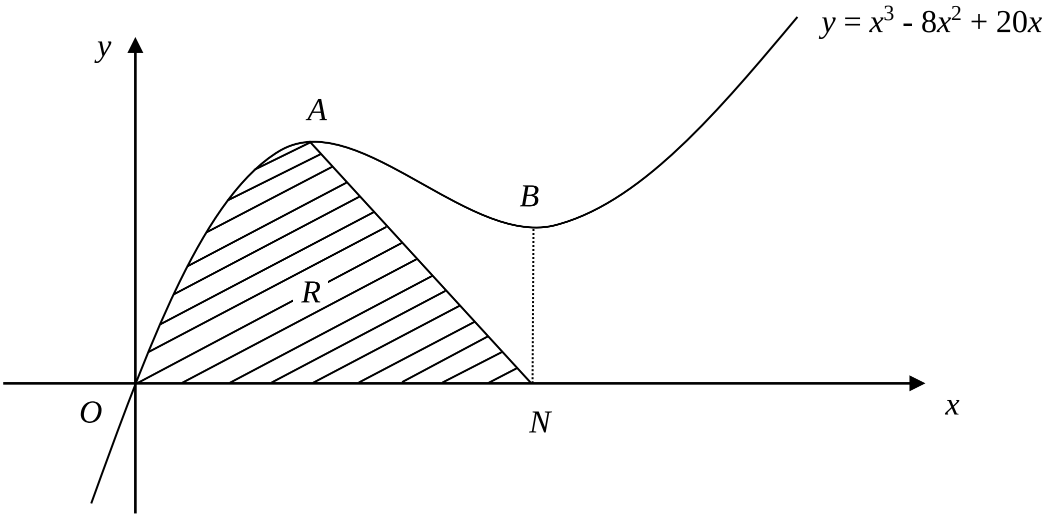


Figure 3 shows a sketch of part of the curve with equation $y = x^3 - 8x^2 + 20x$. The curve has stationary points A and B.

(a) Use calculus to find the x-coordinates of A and B.

(4)

(b) Find the value of $\frac{d^2y}{dx^2}$ at A, and hence verify that A is a maximum.

(2)

The line through B parallel to the y-axis meets the x-axis at the point N. The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line from A to N.

(c) Find $\int (x^3 - 8x^2 + 20x) \; dx$.

(3)

(d) Hence calculate the exact area of R.

(5)

$$f(x) = x^3 + 3x^2 + 5.$$

Find

(b) $\int_1^2 f(x) \; dx$.

(4)

Figure 1

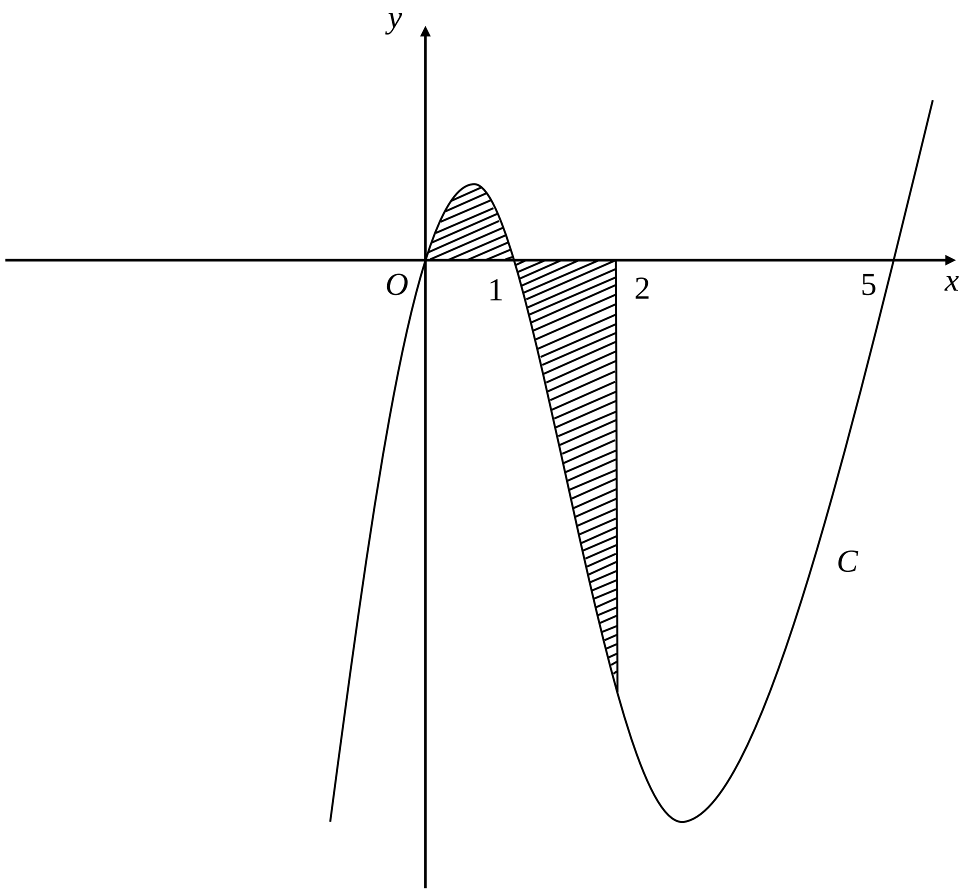


Figure 1 shows a sketch of part of the curve C with equation

$$y = x(x - 1)(x - 5).$$

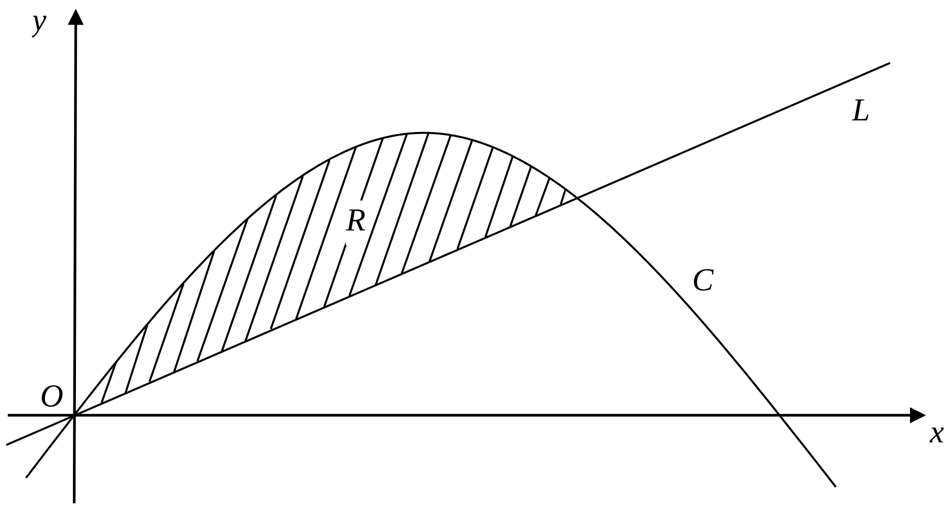
Use calculus to find the total area of the finite region, shown shaded in Figure 1, that is between $x = 0$ and $x = 2$ and is bounded by C, the x-axis and the line $x = 2$.

(9)

Evaluate $\int_1^8 \frac{1}{\sqrt{x}}\,dx$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers.

(4)

Figure 2



In Figure 2 the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects with the x-axis at $x = 0$ and $x = 6$.

(1)

(b) Show that the line L intersects the curve C at the points (0, 0) and (4, 8).

(3)

The region R, bounded by the curve C and the line L, is shown shaded in Figure 2.

(c) Use calculus to find the area of R.

(6)

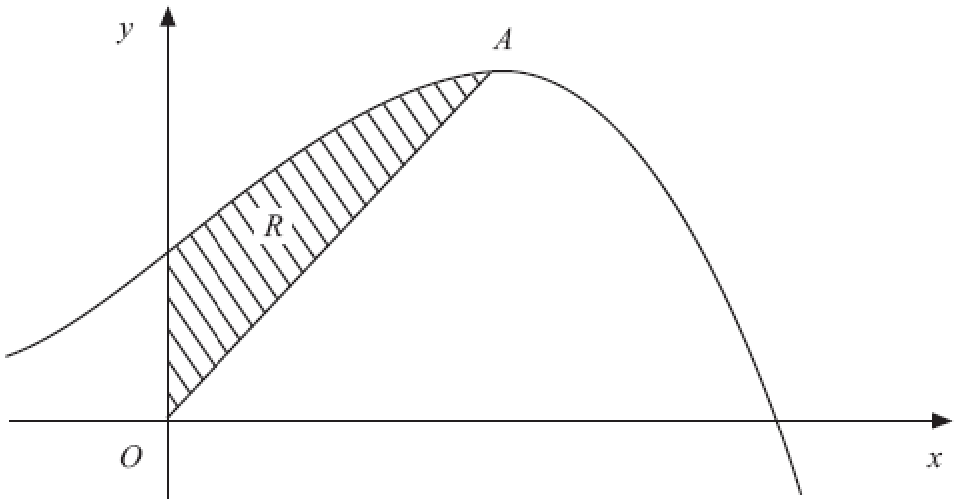


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 10 + 8x + x^2 - x^3$.

The curve has a maximum turning point A.

(a) Using calculus, show that the x-coordinate of A is 2.

(3)

The region R, shown shaded in Figure 2, is bounded by the curve, the y-axis and the line from O to A, where O is the origin.

(b) Using calculus, find the exact area of R.

(8)

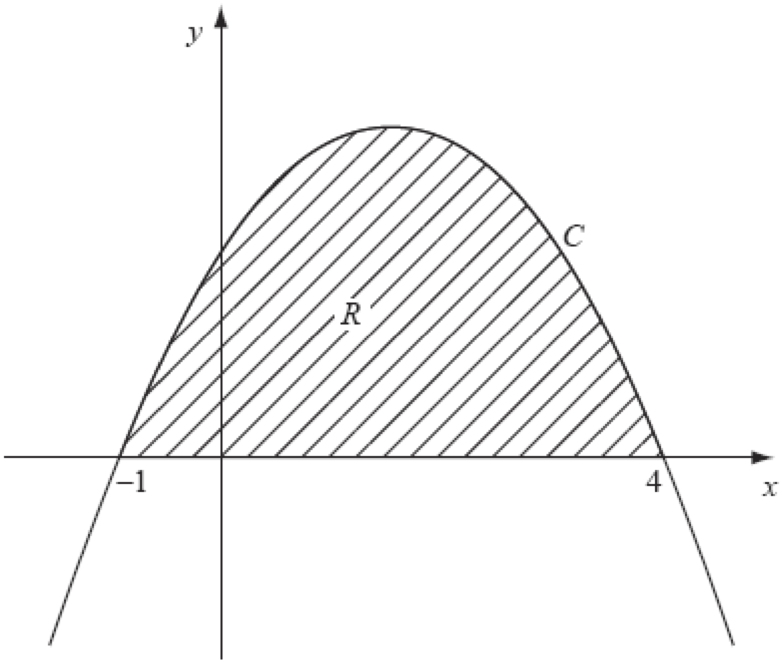


Figure 1

Figure 1 shows part of the curve C with equation $y = (1 + x)(4 - x)$.

The curve intersects the x-axis at $x = -1$ and $x = 4$. The region R, shown shaded in Figure 1, is bounded by C and the x-axis.

Use calculus to find the exact area of R.

(5)

Use calculus to find the value of

$$\int_1^4 (2x + 3\sqrt{x}) \, dx.$$

(5)

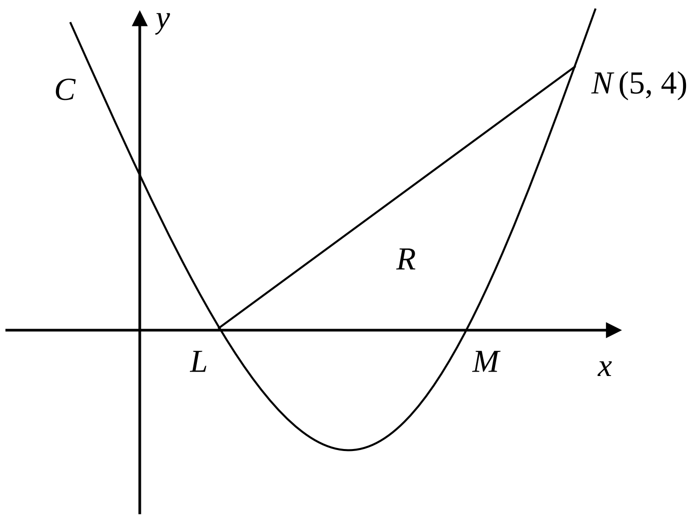


Figure 2

The curve C has equation $y = x^2 - 5x + 4$. It cuts the x-axis at the points L and M as shown in Figure 2.

(a) Find the coordinates of the point L and the point M.

(2)

(b) Show that the point $N(5, 4)$ lies on C.

(1)

(c) Find $\int (x^2 - 5x + 4)\ \mathrm{d}x$.

(2)

The finite region R is bounded by LN, LM and the curve C as shown in Figure 2.

(d) Use your answer to part (c) to find the exact value of the area of R.

(5)

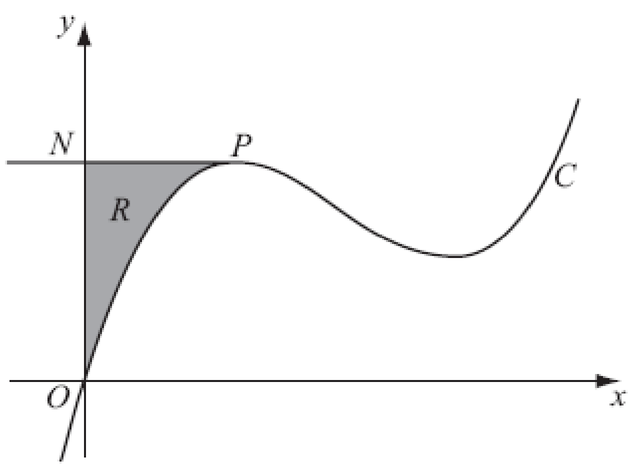


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(a) show that $k = 28$.

(3)

The line through P parallel to the x-axis cuts the y-axis at the point N.
The region R is bounded by C, the y-axis and PN, as shown shaded in Figure 2.

(b) Use calculus to find the exact area of R.

(6)

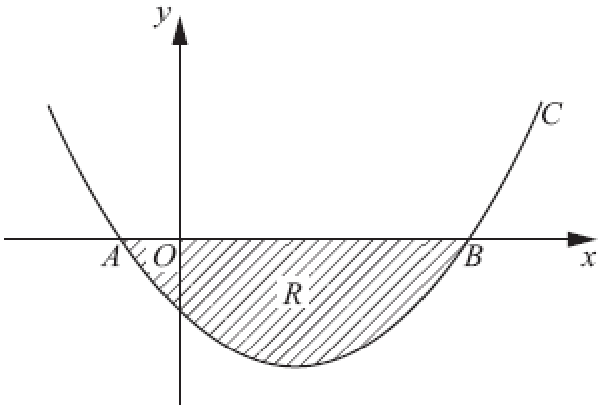


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x + 1)(x - 5).$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B.

(1)

The finite region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Use integration to find the area of R.

(6)

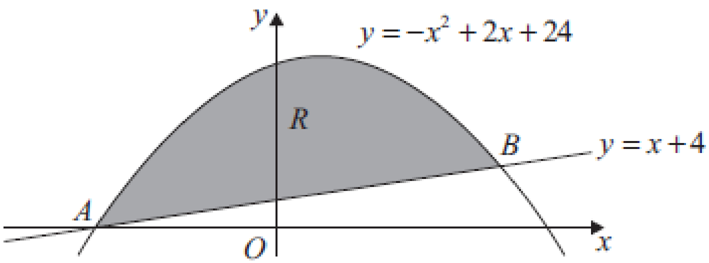


Figure 3

The straight line with equation $y = x + 4$ cuts the curve with equation $y = -x^2 + 2x + 24$ at the points A and B, as shown in Figure 3.

(a) Use algebra to find the coordinates of the points A and B.

(4)

The finite region R is bounded by the straight line and the curve and is shown shaded in Figure 3.

(b) Use calculus to find the exact area of R.

(7)

(a) Find $\int \left(3 + 4x^3 - \frac{2}{x^2}\right) dx$.

(3)

(b) Hence evaluate $\int_1^2 \left(3 + 4x^3 - \frac{2}{x^2}\right) dx$. **(2)**

Figure 2

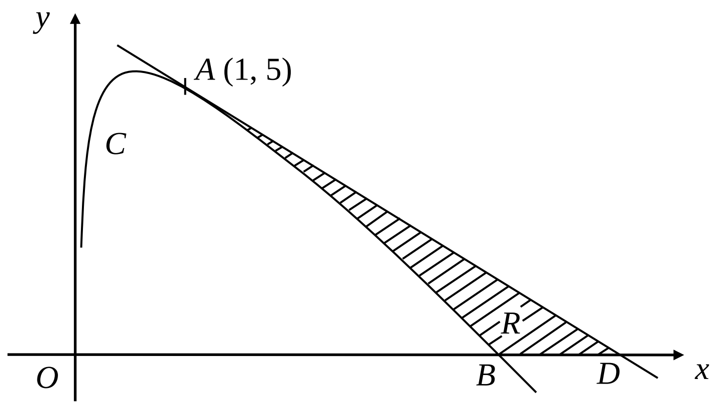


Figure 2 shows part of the curve C with equation

$$y = 9 - 2x - \frac{2}{\sqrt{x}}, \qquad x > 0.$$

The point $A(1, 5)$ lies on C and the curve crosses the x-axis at $B(b, 0)$, where b is a constant and $b > 0$.

(a) Verify that $b = 4$.

(1)

The tangent to C at the point A cuts the x-axis at the point D, as shown in Fig. 2.

(b) Show that an equation of the tangent to C at A is $y + x = 6$.

(4)

(c) Find the coordinates of the point D.

(1)

The shaded region R, shown in Fig. 2, is bounded by C, the line AD and the x-axis.

(d) Use integration to find the area of R.

(6)