Pure Mathematics - Algebra

Do not use a calculator in this test

1. Solve each of the following quadratic equations, if possible, giving answers in exact form.
   (i) $2x^2 - x - 3 = 0$
   (ii) $3x^2 - 2x + 4 = 0$
   (iii) $x^2 + 5x - 1 = 0$ [8]

2. (i) Write the quadratic expression $x^2 + 4x + 5$ in the form $A(x + B)^2 + C$. [3]

   (ii) Hence write down the coordinates of the minimum point of the graph $y = x^2 + 2x + 5$. [2]
   (iii) Find the discriminant of the quadratic equation $x^2 + 4x + 5 = 0$. [2]
   (iv) What does the value of this discriminant tell you about the solutions of the equation $x^2 + 4x + 5 = 0$? [1]
   (v) Sketch the graph of $y = x^2 + 4x + 5$, and explain how this confirms your answer to (iv). [3]

3. The quadratic equation $2x^2 + 5x + k = 0$ has equal roots.
   (i) Find the value of k. [3]
   (ii) Solve the equation $2x^2 + 5x + k = 0$. [3]

4. (i) Write the expression $2x^2 + 2x - 1$ in the form $a(x + p)^2 + q$. [4]
   (ii) Hence, or otherwise, solve the equation $2x^2 + 2x - 1 = 0$. [3]

5. Solve the following inequalities.
   (i) $2x + 3 < 1 - x$ [3]
   (ii) $3(y - 1) \geq 5y - 8$ [3]

6. Solve the following inequalities.
   (i) $x^2 + 2x - 15 \leq 0$ [4]
   (ii) $2p^2 - 7p + 3 > 0$ [4]
   (iii) $z(2 - z) < z - 12$ [5]

7. Find the coordinates of the points where the graphs of $x + 2y = 13$ and $x^2 - y^2 = 9$ intersect. [7]

8. (i) Add $(x^3 + 2x^2 - 3x + 1)$ to $(2x^3 + 5x - 3)$ [2]
   (ii) Subtract $(2x^3 - 3x^2 + x - 2)$ from $(x^4 + x^3 - 2x^2 + 1)$ [2]
   (iii) Multiply $(x^3 + 4x^2 - 2x + 3)$ by $(2x - 1)$ [4]
   (iv) Multiply $(x^2 + 2x + 3)$ by $(x^2 - x + 1)$ [4]

Total 70 marks

Algebra Solutions

Solutions

1. **(i)** $2x^2 - x - 3 = 0$

$a = 2, b = -1, c = -3$

Discriminant $= b^2 - 4ac = (-1)^2 - 4 \times 2 \times -3 = 1 + 24 = 25$

Since the discriminant is a perfect square, the equation can be factorised.

$(2x - 3)(x + 1) = 0$

$x = \frac{3}{2}$ or $x = -1$

(ii) $3x^2 - 2x + 4 = 0$

$a = 3, b = -2, c = 4$

Discriminant $= b^2 - 4ac = (-2)^2 - 4 \times 3 \times 4 = 4 - 48 = -44$

The discriminant is negative, so the equation has no real solution.

(iii) $x^2 + 5x - 1 = 0$

$a = 1, b = 5, c = -1$

Discriminant $= b^2 - 4ac = 5^2 - 4 \times 1 \times -1 = 25 + 4 = 29$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{29}}{2 \times 1} = \frac{-5 \pm \sqrt{29}}{2}$$

2. **(i)** $x^2 + 4x + 5 = (x + 2)^2 - 4 + 5$

$= (x + 2)^2 + 1$

(ii) Minimum point is (-2, 1)

(ii) $x^2 + 4x + 5 = 0$

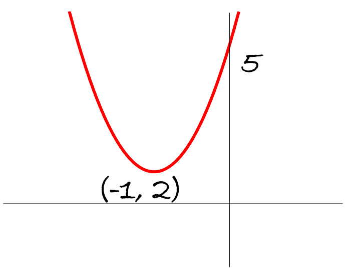
$a = 1, b = 4, c = 5$

Discriminant $= b^2 - 4ac = 4^2 - 4 \times 1 \times 5 = 16 - 20 = -4$

(iii) Since the discriminant is negative, there are no real solutions to to the equation $x^2 + 4x + 5 = 0$.

(iv) The minimum point is above the x-ax ... ot cut the x-axis and therefore there are no sol

$x^2 + 4x + 5 = 0$.



Algebra Solutions

3. **(i)** $2x^2+5x+k=0$

$a=2, b=5, c=k$

If roots are equal, $b^2-4ac=0$

$5^2-4\times 2\times k=0$

$8k=25$

$k=\dfrac{25}{8}$

(ii) $2x^2+5x+\frac{25}{8}=0$

$16x^2+40x+25=0$

$(4x+5)^2=0$

$x=-\frac{5}{4}$

4. **(i)** $2x^2+2x-1=2\left(x^2+x\right)-1$

$=2\left(\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2\right)-1$

$=2\left(x+\frac{1}{2}\right)^2-2\times\frac{1}{4}-1$

$=2\left(x+\frac{1}{2}\right)^2-\frac{1}{2}-1$

$=2\left(x+\frac{1}{2}\right)^2-\frac{3}{2}$

(ii) $2x^2+2x-1=0$

$2\left(x+\frac{1}{2}\right)^2-\frac{3}{2}=0$

$2\left(x+\frac{1}{2}\right)^2=\frac{3}{2}$

$\left(x+\frac{1}{2}\right)^2=\frac{3}{4}$

$x+\frac{1}{2}=\pm\frac{\sqrt{3}}{2}$

$x=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}$

5. **(i)** $2x+3<1-x$

$3x<-2$

$x<-\frac{2}{3}$

Algebra Solutions

(ii) $3(y-1) < 5y-8$

$3y-3 < 5y-8$

$5 < 2y$

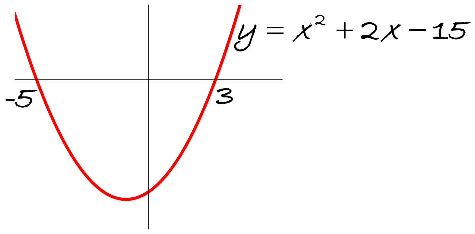
$2y > 5$

$y > \frac{5}{2}$

6. **(i)** $x^2+2x-15 \le 0$

$(x+5)(x-3) \le 0$

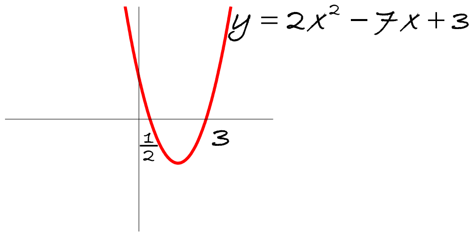
From graph, $-5 \le x \le 3$



(ii) $2p^2-7p+3 > 0$

$(2p-1)(p-3) > 0$

From graph, $p < \frac{1}{2}$ or $p > 3$.



(iii) $z(2-z) < z-12$

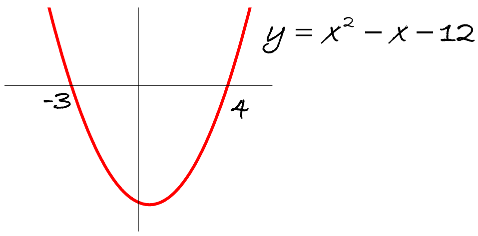
$2z-z^2 < z-12$

$0 < z^2-z-12$

$z^2-z-12 > 0$

$(z-4)(z+3) > 0$

From graph, $z < -3$ or $z > 4$.



7. $x+2y=13 \qquad (1)$

$x^2-y^2=9 \qquad (2)$

(1) $\Rightarrow x = 13-2y$

Substituting into (2): $(13-2y)^2-y^2=9$

$169-52y+4y^2-y^2=9$

$3y^2-52y+160=0$

$(y-4)(3y-40)=0$

$y=4$ or $y=\frac{40}{3}$

When $y=4$, $x=13-8=5$

When $y=\frac{40}{3}$, $x=13-\frac{80}{3}=-\frac{41}{3}$

The points of intersection are (5, 4) and $\left(-\frac{41}{3}, \frac{40}{3}\right)$.

Algebra Solutions

8. **(i)** $(x^3+2x^2-3x+1)+(2x^3+5x-3)$

$$=x^3+2x^3+2x^2-3x+5x+1-3$$
$$=3x^3+2x^2+2x-2$$

(ii) $(x^4+x^3-2x^2+1)-(2x^3-3x^2+x-2)$

$$=x^4+x^3-2x^2+1-2x^3+3x^2-x+2$$
$$=x^4+x^3-2x^3-2x^2+3x^2-x+1+2$$
$$=x^4-x^3+x^2-x+3$$

(iii) $(2x-1)(x^3+4x^2-2x+3)$

$$=2x(x^3+4x^2-2x+3)-(x^3+4x^2-2x+3)$$
$$=2x^4+8x^3-4x^2+6x-x^3-4x^2+2x-3$$
$$=2x^4+8x^3-x^3-4x^2-4x^2+6x+2x-3$$
$$=2x^4+7x^3-8x^2+8x-3$$

(iv) $(x^2+2x+3)(x^2-x+1)$

$$=x^2(x^2-x+1)+2x(x^2-x+1)+3(x^2-x+1)$$
$$=x^4-x^3+x^2+2x^3-2x^2+2x+3x^2-3x+3$$
$$=x^4-x^3+2x^3+x^2-2x^2+3x^2+2x-3x+3$$
$$=x^4-x^3+2x^2-x+3$$