Revision Notes for Core 3

Functions

Domain is the set of inputs for a function. Can be all real numbers or only a particular few.

Range is the set of output numbers obtained from the domain. Use completing the square to find the lowest and highest values for the range if a quadratic.

Composite functions

gf(x) = g(f(x)) means substitute in x into function f then substitute in that result into function g. It is a function of a function.

Inverse function means just doing the function in reverse

$f^{-1}(x)$ when f(x) = (2x+3)/5 becomes (5x-3)/2. Think of as a flow diagram followed in reverse. The range and domain swap around when doing the inverse and it is a reflection in the line y=x if they are one-to-one functions (one value for the range for one value from the domain)

Differentiating $(ax+b)^n$

Use the chain rule

dy/dx = dy/du x du/dx where u=ax+b Use whenever you have a function of a function. Diff one function then diff the other and multiply the results together. Use for rates of change as well to find dy/dt for instance.

Product Rule

If y=uv then $\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

$\frac{dy}{dx}$ = (**diff leave + leave diff**)

e.g $\frac{d}{dx}x^2\ln x = 2x\ln x + x^2\frac{1}{x}$

Quotient Rule

If $y=\frac{u}{v}$ then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Make sure u is the numerator and v the denominator.

Integrating $(ax+b)^n$

$\int (ax+b)^n dx = \frac{1}{n+1} \times \frac{1}{a}(ax+b)^{n+1} + c$

This is because if you differentiate $(ax+b)^{n+1}$ you get: a x (n+1) $(ax+b)^n$ so the $\frac{1}{n+1} \times \frac{1}{a}$ counteracts this.

Differentiating and integrating e^x and ln x

$\frac{d}{dx}e^x = e^x$ as by definition the gradient is the same as the function and therefore

$\int e^x dx = e^x + c$ $\frac{d}{dx}e^{3x}$ use chain rule so = $3e^{3x}$ and $\int e^{3x} dx = \frac{1}{3}e^{3x} + c$ for the same reason as the reverse of the chain rule.

Log Laws

$\ln a + \ln b = \ln ab$ $\ln a - \ln b = \ln \frac{a}{b}$ $\ln 1 = 0$ $\ln a^x = x \ln a$

Remember that if $y = e^x$ then if I take the natural log (ln) of both sides, then x = ln y (e^x and ln x are the inverse of each other).   **LEARN: $y = e^x$ so $x = \ln y$**

$\frac{d}{dx}\ln x = \frac{1}{x}$ and $\int \frac{1}{x} dx = \ln x + c$

If you can take out a coefficient then do $\int \frac{1}{3x} dx = \frac{1}{3}\int \frac{1}{x} dx = \frac{1}{3}\ln x + c$

If of the form $\int \frac{1}{ax+b} dx$ then $= \frac{1}{a}\ln(ax+b)$ as $\frac{1}{a}$ counteracts the differentiation of ax+b.

Trigonometry

Learn the formulae

$\sec x = \frac{1}{\cos x}$ $\quad$ $\mathrm{cosec}\, x = \frac{1}{\sin x}$ $\quad$ $\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$ $\quad$ $1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \mathrm{cosec}^2 x$ $\quad$ $\sin 2A = 2\sin A\cos A$ $\quad$ $\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$ $\quad$ $1 + \cos 2A = 2\cos^2 A$ $\quad$ $1 - \cos 2A = 2\sin^2 A$

The formulae below are in the booklet you get in the exam so no need to learn:

$\sin(A \pm B) = \sin A\cos B \pm \sin B\cos A$ $\quad$ $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$a\sin x \pm b\cos x$ can be written as $R\sin(x \pm \alpha)$ or $R\cos(x \pm \alpha)$ where $R = \sqrt{(a^2 + b^2)}$ and the angle α is found by using the addition formula for $\sin(A\pm B)$ or $\cos(A \mp B)$.

Modulus

Always positive. $|x| = x$ and $|-x| = x$. Graph of $y=|f(x)|$ will not go below the x-axis. It is $y=f(x)$ but with the negative y bits being reflected in the x-axis.

If solving modulus questions, square both sides first then solve the quadratic.

Solving equations using the sign change rule

When the curve crosses the x-axis, f(x) will change from negative to positive or vice versa. Use a decimal search to get the solution to the right degree of accuracy.

Use upper and lower bounds to test that the root is correct to that degree of accuracy.

Can use differentiation to find the turning points and plot the graph, then see what values of x the root lies between.

Iteration

Rearrange equation to give $x_{r+1}=f(x_r)$… then sub in x-values for x_r to give x_{r+1} and sub in again. Repeat into the solution converges to a limit. If doesn't converge, rearrange the equation the other way round and try again.

Volumes of Revolution

f(x) gives the radius (y-value) so the area is $\pi(f(x))^2$ and the volume is the integral of this between the 2 limits (x=a and x=b). (π can go outside the integral sign as a constant.)

$$\pi \int_a^b (f(x))^2 \, dx$$

If the volume is between the curve, y-axis and y=a and y=b, then rearrange to give f(y)

e.g If $y=2x^3$ then $x=(\frac{y}{2})^{1/3}$ so this is f(y).

If you want the volume of a region between 2 curves, you can calculate the volumes separately then subtract or combine the equations first (f(x)- g(x)) then integrate this result.

Simpson's Rule

Used to find an approximate answer to the area under a curve if you can't integrate it.

Approximates a curve to a quadratic needing 3 points on the curve to solve.

h=width of each interval (Must have an even number of strips.)

$$\int_a^b f(x)dx = \tfrac{1}{3}h(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \ldots + y_{2n}) = \tfrac{1}{3}h(y_{ends} + 4y_{odds} + 2y_{evens})$$

Transformations

Replace x with x – k then it's a translation of k units in the x-direction.

Replace y with y – k then it's a translation of k units in the y-direction.

Replace x with $\frac{x}{k}$ then it's a stretch of factor k in the x-direction.

Replace y with $\frac{y}{k}$ then it's a stretch of factor k in the y-direction.

Replace x with -x then it's a reflection in the y-axis. (***Remember IT'S THE OPPOSITE***

Replace y with -y then it's a reflection in the x-axis. $\frac{x}{k}$ ***is really a k times stretch in x***)