The Grange School
Maths Department

Core 2
OCR Past Papers

1 A sequence S has terms $u_1, u_2, u_3, \ldots$ defined by

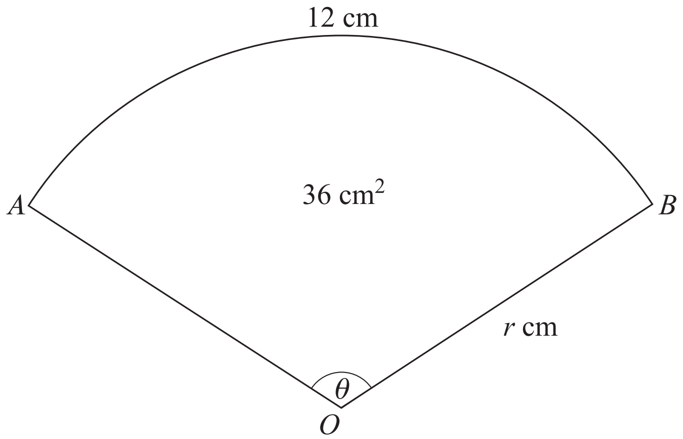
$$u_n = 3n - 1,$$

for $n \geqslant 1$.

(i) Write down the values of u_1, u_2 and u_3, and state what type of sequence S is. [3]

(ii) Evaluate $\sum_{n=1}^{100} u_n$. [3]

2



A sector OAB of a circle of radius r cm has angle θ radians. The length of the arc of the sector is 12 cm and the area of the sector is 36 cm^2 (see diagram).

(i) Write down two equations involving r and θ. [2]

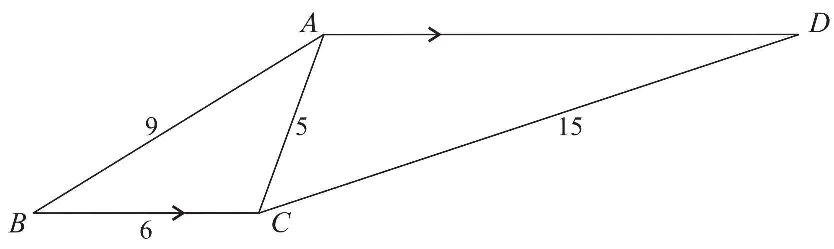
(ii) Hence show that $r = 6$, and state the value of θ. [2]

(iii) Find the area of the segment bounded by the arc AB and the chord AB. [3]

3 **(i)** Find $\int (2x + 1)(x + 3)\,dx$. [4]

(ii) Evaluate $\int_0^9 \frac{1}{\sqrt{x}}\,dx$. [3]

4



In the diagram, $ABCD$ is a quadrilateral in which AD is parallel to BC. It is given that $AB = 9$, $BC = 6$, $CA = 5$ and $CD = 15$.

(i) Show that $\cos BCA = -\frac{1}{3}$, and hence find the value of $\sin BCA$. [4]

(ii) Find the angle ADC correct to the nearest $0.1°$. [4]

5 The cubic polynomial $f(x)$ is given by

$$f(x) = x^3 + ax + b,$$

where a and b are constants. It is given that $(x + 1)$ is a factor of $f(x)$ and that the remainder when $f(x)$ is divided by $(x - 3)$ is 16.

(i) Find the values of a and b. [5]

(ii) Hence verify that $f(2) = 0$, and factorise $f(x)$ completely. [3]

6 **(i)** Find the binomial expansion of $\left(x^2 + \frac{1}{x}\right)^3$, simplifying the terms. [4]

(ii) Hence find $\int \left(x^2 + \frac{1}{x}\right)^3 \mathrm{d}x$. [4]

7 **(i)** Evaluate $\log_5 15 + \log_5 20 - \log_5 12$. [3]

(ii) Given that $y = 3 \times 10^{2x}$, show that $x = a \log_{10}(by)$, where the values of the constants a and b are to be found. [4]

[Questions 8 and 9 are printed overleaf.]

8 The amounts of oil pumped from an oil well in each of the years 2001 to 2004 formed a geometric progression with common ratio 0.9. The amount pumped in 2001 was 100 000 barrels.

(i) Calculate the amount pumped in 2004. [2]

It is assumed that the amounts of oil pumped in future years will continue to follow the same geometric progression. Production from the well will stop at the end of the first year in which the amount pumped is less than 5000 barrels.

(ii) Calculate in which year the amount pumped will fall below 5000 barrels. [4]

(iii) Calculate the total amount of oil pumped from the well from the year 2001 up to and including the final year of production. [3]

9 **(a)** **(i)** Write down the exact values of $\cos \frac{1}{6}\pi$ and $\tan \frac{1}{3}\pi$ (where the angles are in radians). Hence verify that $x = \frac{1}{6}\pi$ is a solution of the equation

$$2 \cos x = \tan 2x. \quad [3]$$

(ii) Sketch, on a single diagram, the graphs of $y = 2 \cos x$ and $y = \tan 2x$, for x (radians) such that $0 \leqslant x \leqslant \pi$. Hence state, in terms of π, the other values of x between 0 and π satisfying the equation

$$2 \cos x = \tan 2x. \quad [4]$$

(b) **(i)** Use the trapezium rule, with 3 strips, to find an approximate value for the area of the region bounded by the curve $y = \tan x$, the x-axis, and the lines $x = 0.1$ and $x = 0.4$. (Values of x are in radians.) [4]

(ii) State with a reason whether this approximation is an underestimate or an overestimate. [1]

1 The 20th term of an arithmetic progression is 10 and the 50th term is 70.

(i) Find the first term and the common difference. [4]

(ii) Show that the sum of the first 29 terms is zero. [2]

2 Triangle ABC has $AB = 10$ cm, $BC = 7$ cm and angle $B = 80°$. Calculate

(i) the area of the triangle, [2]

(ii) the length of CA, [2]

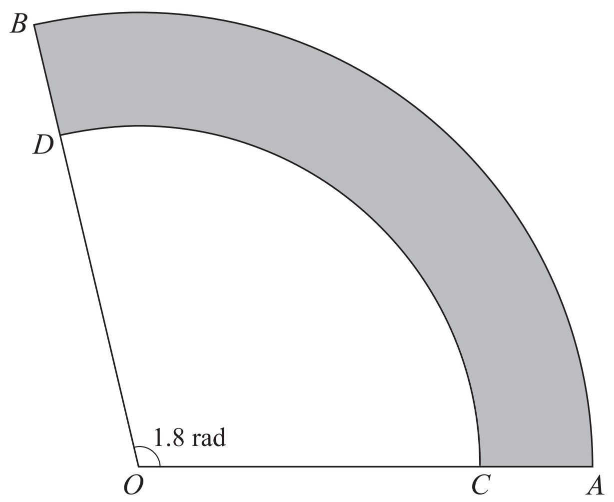
(iii) the size of angle C. [2]

3 **(i)** Find the first three terms of the expansion, in ascending powers of x, of $(1 - 2x)^{12}$. [3]

(ii) Hence find the coefficient of x^2 in the expansion of

$$(1 + 3x)(1 - 2x)^{12}.$$ [3]

4



The diagram shows a sector OAB of a circle with centre O. The angle AOB is 1.8 radians. The points C and D lie on OA and OB respectively. It is given that $OA = OB = 20$ cm and $OC = OD = 15$ cm. The shaded region is bounded by the arcs AB and CD and by the lines CA and DB.

(i) Find the perimeter of the shaded region. [3]

(ii) Find the area of the shaded region. [3]

5 In a geometric progression, the first term is 5 and the second term is 4.8.

(i) Show that the sum to infinity is 125. [2]

(ii) The sum of the first n terms is greater than 124. Show that

$$0.96^n < 0.008,$$

and use logarithms to calculate the smallest possible value of n. [6]

6 **(a)** Find $\int \left(x^{\frac{1}{2}} + 4\right) \mathrm{d}x$. [4]

(b) **(i)** Find the value, in terms of a, of $\int_1^a 4x^{-2}\,\mathrm{d}x$, where a is a constant greater than 1. [3]

(ii) Deduce the value of $\int_1^\infty 4x^{-2}\,\mathrm{d}x$. [1]

7 **(i)** Express each of the following in terms of $\log_{10} x$ and $\log_{10} y$.

(a) $\log_{10}\left(\frac{x}{y}\right)$ [1]

(b) $\log_{10}(10x^2y)$ [3]

(ii) Given that

$$2\log_{10}\left(\frac{x}{y}\right) = 1 + \log_{10}(10x^2y),$$

find the value of y correct to 3 decimal places. [4]

8 The cubic polynomial $2x^3 + kx^2 - x + 6$ is denoted by $\mathrm{f}(x)$. It is given that $(x + 1)$ is a factor of $\mathrm{f}(x)$.

(i) Show that $k = -5$, and factorise $\mathrm{f}(x)$ completely. [6]

(ii) Find $\int_{-1}^{2} \mathrm{f}(x)\,\mathrm{d}x$. [4]

(iii) Explain with the aid of a sketch why the answer to part **(ii)** does not give the area of the region between the curve $y = \mathrm{f}(x)$ and the x-axis for $-1 \leqslant x \leqslant 2$. [2]

[Question 9 is printed overleaf.]

9 **(i)** Sketch, on a single diagram showing values of x from $-180°$ to $+180°$, the graphs of $y = \tan x$ and $y = 4\cos x$. [3]

The equation

$$\tan x = 4\cos x$$

has two roots in the interval $-180° \leqslant x \leqslant 180°$. These are denoted by α and β, where $\alpha < \beta$.

(ii) Show α and β on your sketch, and express β in terms of α. [3]

(iii) Show that the equation $\tan x = 4\cos x$ may be written as

$$4\sin^2 x + \sin x - 4 = 0.$$

Hence find the value of $\beta - \alpha$, correct to the nearest degree. [6]

1 Find the binomial expansion of $(3x-2)^4$. [4]

2 A sequence of terms $u_1, u_2, u_3, \ldots$ is defined by

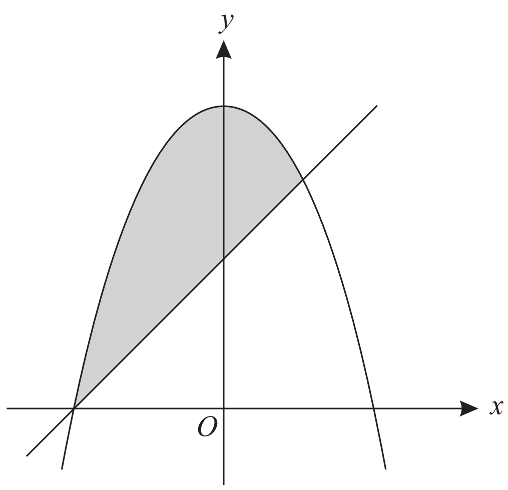
$$u_1 = 2 \qquad \text{and} \qquad u_{n+1} = 1 - u_n \text{ for } n \geqslant 1.$$

(i) Write down the values of u_2, u_3 and u_4. [2]

(ii) Find $\sum_{n=1}^{100} u_n$. [3]

3 The gradient of a curve is given by $\frac{dy}{dx} = 2x^{-\frac{1}{2}}$, and the curve passes through the point (4, 5). Find the equation of the curve. [6]

4



The diagram shows the curve $y = 4 - x^2$ and the line $y = x + 2$.

(i) Find the x-coordinates of the points of intersection of the curve and the line. [2]

(ii) Use integration to find the area of the shaded region bounded by the line and the curve. [6]

5 Solve each of the following equations, for $0° \leqslant x \leqslant 180°$.

(i) $2\sin^2 x = 1 + \cos x$. [4]

(ii) $\sin 2x = -\cos 2x$. [4]

6 **(i)** John aims to pay a certain amount of money each month into a pension fund. He plans to pay £100 in the first month, and then to increase the amount paid by £5 each month, i.e. paying £105 in the second month, £110 in the third month, etc.

If John continues making payments according to this plan for 240 months, calculate

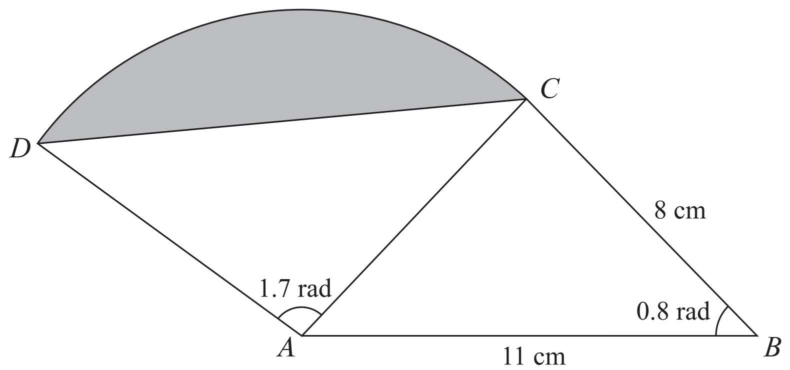
(a) how much he will pay in the final month, [2]

(b) how much he will pay altogether over the whole period. [2]

(ii) Rachel also plans to pay money monthly into a pension fund over a period of 240 months, starting with £100 in the first month. Her monthly payments will form a geometric progression, and she will pay £1500 in the final month.

Calculate how much Rachel will pay altogether over the whole period. [5]

7



The diagram shows a triangle ABC, and a sector ACD of a circle with centre A. It is given that $AB = 11$ cm, $BC = 8$ cm, angle $ABC = 0.8$ radians and angle $DAC = 1.7$ radians. The shaded segment is bounded by the line DC and the arc DC.

(i) Show that the length of AC is 7.90 cm, correct to 3 significant figures. [3]

(ii) Find the area of the shaded segment. [3]

(iii) Find the perimeter of the shaded segment. [4]

8 The cubic polynomial $2x^3 + ax^2 + bx - 10$ is denoted by $f(x)$. It is given that, when $f(x)$ is divided by $(x - 2)$, the remainder is 12. It is also given that $(x + 1)$ is a factor of $f(x)$.

(i) Find the values of a and b. [6]

(ii) Divide $f(x)$ by $(x + 2)$ to find the quotient and the remainder. [5]

[Question 9 is printed overleaf.]

9 **(i)** Sketch the curve $y = \left(\frac{1}{2}\right)^x$, and state the coordinates of any point where the curve crosses an axis. [3]

(ii) Use the trapezium rule, with 4 strips of width 0.5, to estimate the area of the region bounded by the curve $y = \left(\frac{1}{2}\right)^x$, the axes, and the line $x = 2$. [4]

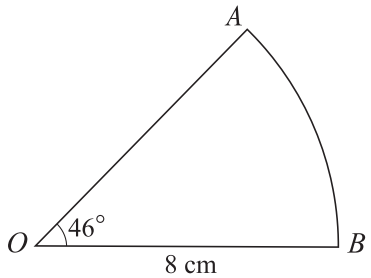
(iii) The point P on the curve $y = \left(\frac{1}{2}\right)^x$ has y-coordinate equal to $\frac{1}{6}$. Prove that the x-coordinate of P may be written as

$$1 + \frac{\log_{10} 3}{\log_{10} 2}.$$ [4]

Jan 2007

1 In an arithmetic progression the first term is 15 and the twentieth term is 72. Find the sum of the first 100 terms. [4]

2



The diagram shows a sector OAB of a circle, centre O and radius 8 cm. The angle AOB is 46°.

(i) Express 46° in radians, correct to 3 significant figures. [2]

(ii) Find the length of the arc AB. [1]

(iii) Find the area of the sector OAB. [2]

3 **(i)** Find $\int (4x - 5)\,dx$. [2]

(ii) The gradient of a curve is given by $\frac{dy}{dx} = 4x - 5$. The curve passes through the point (3, 7). Find the equation of the curve. [3]

4 In a triangle ABC, $AB = 5\sqrt{2}$ cm, $BC = 8$ cm and angle $B = 60°$.

(i) Find the exact area of the triangle, giving your answer as simply as possible. [3]

(ii) Find the length of AC, correct to 3 significant figures. [3]

5 **(a)** **(i)** Express $\log_3(4x + 7) - \log_3 x$ as a single logarithm. [1]

(ii) Hence solve the equation $\log_3(4x + 7) - \log_3 x = 2$. [3]

(b) Use the trapezium rule, with two strips of width 3, to find an approximate value for

$$\int_3^9 \log_{10} x \, dx,$$

giving your answer correct to 3 significant figures. [4]

6 **(i)** Find and simplify the first four terms in the expansion of $(1+4x)^7$ in ascending powers of x. [4]

(ii) In the expansion of

$$(3+ax)(1+4x)^7,$$

the coefficient of x^2 is 1001. Find the value of a. [3]

7 **(i) (a)** Sketch the graph of $y = 2\cos x$ for values of x such that $0° \leqslant x \leqslant 360°$, indicating the coordinates of any points where the curve meets the axes. [2]

(b) Solve the equation $2\cos x = 0.8$, giving all values of x between $0°$ and $360°$. [3]

(ii) Solve the equation $2\cos x = \sin x$, giving all values of x between $-180°$ and $180°$. [3]

8 The polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = x^3 - 9x^2 + 7x + 33$.

(i) Find the remainder when $\mathrm{f}(x)$ is divided by $(x+2)$. [2]

(ii) Show that $(x-3)$ is a factor of $\mathrm{f}(x)$. [1]

(iii) Solve the equation $\mathrm{f}(x) = 0$, giving each root in an exact form as simply as possible. [6]

9 On its first trip between Malby and Grenlish, a steam train uses 1.5 tonnes of coal. As the train does more trips, it becomes less efficient so that each subsequent trip uses 2% more coal than the previous trip.

(i) Show that the amount of coal used on the fifth trip is 1.624 tonnes, correct to 4 significant figures. [2]

(ii) There are 39 tonnes of coal available. An engineer wishes to calculate N, the total number of trips possible. Show that N satisfies the inequality

$$1.02^N \leqslant 1.52.$$ [4]

(iii) Hence, by using logarithms, find the greatest number of trips possible. [4]

[Question 10 is printed overleaf.]

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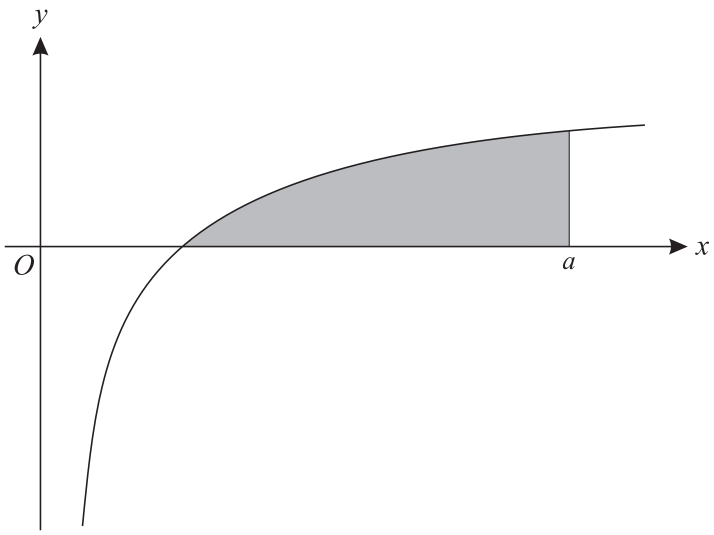
10



The diagram shows the graph of $y = 1 - 3x^{-\frac{1}{2}}$.

(i) Verify that the curve intersects the x-axis at (9, 0). [1]

(ii) The shaded region is enclosed by the curve, the x-axis and the line $x = a$ (where $a > 9$). Given that the area of the shaded region is 4 square units, find the value of a. [9]

1 A geometric progression $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 15 \qquad \text{and} \qquad u_{n+1} = 0.8u_n \quad \text{for } n \geqslant 1.$$

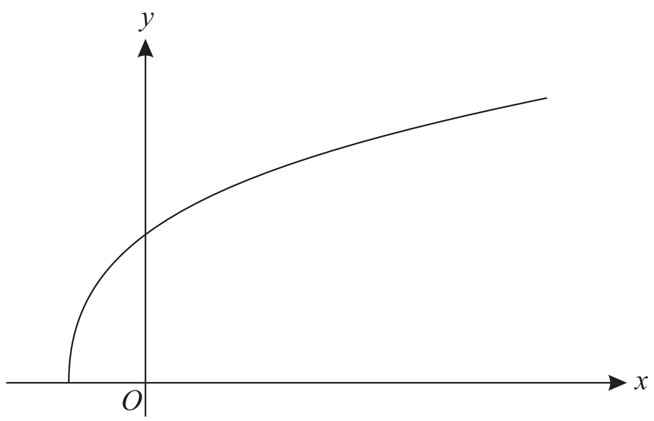
(i) Write down the values of u_2, u_3 and u_4. [2]

(ii) Find $\sum_{n=1}^{20} u_n$. [3]

2 Expand $\left(x + \frac{2}{x}\right)^4$ completely, simplifying the terms. [5]

3 Use logarithms to solve the equation $3^{2x+1} = 5^{200}$, giving the value of x correct to 3 significant figures. [5]

4



The diagram shows the curve $y = \sqrt{4x + 1}$.

(i) Use the trapezium rule, with strips of width 0.5, to find an approximate value for the area of the region bounded by the curve $y = \sqrt{4x + 1}$, the x-axis, and the lines $x = 1$ and $x = 3$. Give your answer correct to 3 significant figures. [4]

(ii) State with a reason whether this approximation is an under-estimate or an over-estimate. [2]

5 **(i)** Show that the equation

$$3\cos^2\theta = \sin\theta + 1$$

can be expressed in the form

$$3\sin^2\theta + \sin\theta - 2 = 0.$$ [2]

(ii) Hence solve the equation

$$3\cos^2\theta = \sin\theta + 1,$$

giving all values of θ between 0° and 360°. [5]

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6 **(a)** **(i)** Find $\int x(x^2 - 4)\,dx$. [3]

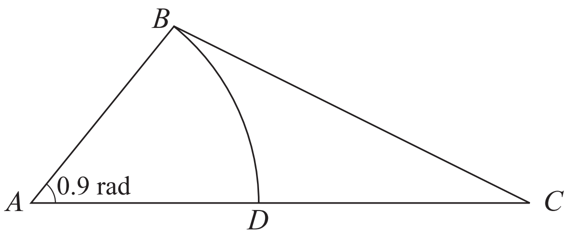
(ii) Hence evaluate $\int_1^6 x(x^2 - 4)\,dx$. [2]

(b) Find $\int \frac{6}{x^3}\,dx$. [3]

7 **(a)** In an arithmetic progression, the first term is 12 and the sum of the first 70 terms is 12 915. Find the common difference. [4]

(b) In a geometric progression, the second term is −4 and the sum to infinity is 9. Find the common ratio. [7]

8



The diagram shows a triangle *ABC*, where angle *BAC* is 0.9 radians. *BAD* is a sector of the circle with centre *A* and radius *AB*.

(i) The area of the sector *BAD* is 16.2 cm^2. Show that the length of *AB* is 6 cm. [2]

(ii) The area of triangle *ABC* is twice the area of sector *BAD*. Find the length of *AC*. [3]

(iii) Find the perimeter of the region *BCD*. [6]

9 The polynomial f(*x*) is given by

$$f(x) = x^3 + 6x^2 + x - 4.$$

(i) **(a)** Show that $(x + 1)$ is a factor of f(*x*). [1]

(b) Hence find the exact roots of the equation f(*x*) = 0. [6]

(ii) **(a)** Show that the equation

$$2\log_2(x + 3) + \log_2 x - \log_2(4x + 2) = 1$$

can be written in the form f(*x*) = 0. [5]

(b) Explain why the equation

$$2\log_2(x + 3) + \log_2 x - \log_2(4x + 2) = 1$$

has only one real root and state the exact value of this root. [2]

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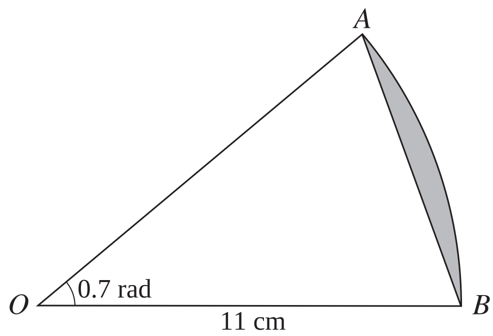
1



The diagram shows a sector AOB of a circle with centre O and radius 11 cm. The angle AOB is 0.7 radians. Find the area of the segment shaded in the diagram. [4]

2 Use the trapezium rule, with 3 strips each of width 2, to estimate the value of

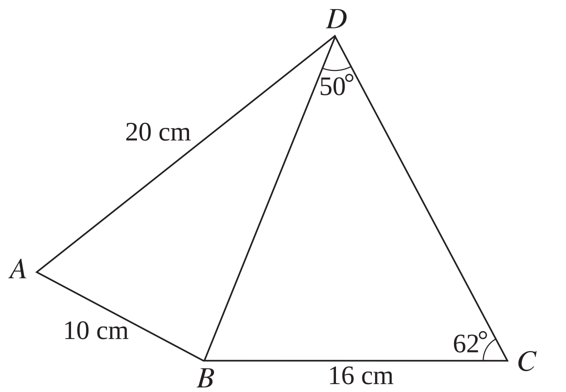
$$\int_1^7 \sqrt{x^2+3}\,dx.$$ [4]

3 Express each of the following as a single logarithm:

(i) $\log_a 2 + \log_a 3$, [1]

(ii) $2\log_{10} x - 3\log_{10} y$. [3]

4



In the diagram, angle $BDC = 50°$ and angle $BCD = 62°$. It is given that $AB = 10$ cm, $AD = 20$ cm and $BC = 16$ cm.

(i) Find the length of BD. [2]

(ii) Find angle BAD. [3]

5 The gradient of a curve is given by $\frac{dy}{dx} = 12\sqrt{x}$. The curve passes through the point (4, 50). Find the equation of the curve. [6]

6 A sequence of terms $u_1, u_2, u_3, \ldots$ is defined by

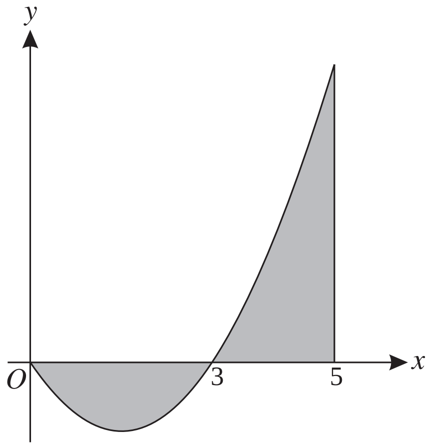
$$u_n = 2n + 5, \quad \text{for } n \geqslant 1.$$

(i) Write down the values of u_1, u_2 and u_3. [2]

(ii) State what type of sequence it is. [1]

(iii) Given that $\sum_{n=1}^{N} u_n = 2200$, find the value of N. [5]

7



The diagram shows part of the curve $y = x^2 - 3x$ and the line $x = 5$.

(i) Explain why $\int_0^5 (x^2 - 3x)\,dx$ does not give the total area of the regions shaded in the diagram. [1]

(ii) Use integration to find the exact total area of the shaded regions. [7]

8 The first term of a geometric progression is 10 and the common ratio is 0.8.

(i) Find the fourth term. [2]

(ii) Find the sum of the first 20 terms, giving your answer correct to 3 significant figures. [2]

(iii) The sum of the first N terms is denoted by S_N, and the sum to infinity is denoted by S_∞.

Show that the inequality $S_\infty - S_N < 0.01$ can be written as

$$0.8^N < 0.0002,$$

and use logarithms to find the smallest possible value of N. [7]

9 **(i)**

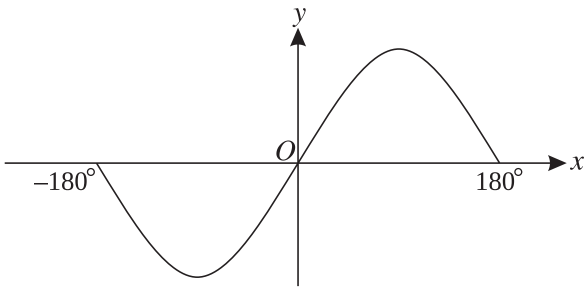


Fig. 1

Fig. 1 shows the curve $y = 2\sin x$ for values of x such that $-180^\circ \leqslant x \leqslant 180^\circ$. State the coordinates of the maximum and minimum points on this part of the curve. [2]

(ii)

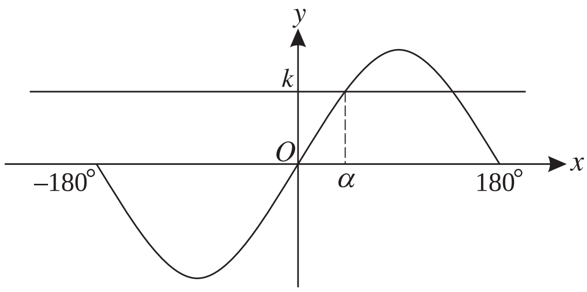


Fig. 2

Fig. 2 shows the curve $y = 2\sin x$ and the line $y = k$. The smallest positive solution of the equation $2\sin x = k$ is denoted by α. State, in terms of α, and in the range $-180^\circ \leqslant x \leqslant 180^\circ$,

(a) another solution of the equation $2\sin x = k$, [1]

(b) one solution of the equation $2\sin x = -k$. [1]

(iii) Find the x-coordinates of the points where the curve $y = 2\sin x$ intersects the curve $y = 2 - 3\cos^2 x$, for values of x such that $-180^\circ \leqslant x \leqslant 180^\circ$. [6]

10 **(i)** Find the binomial expansion of $(2x + 5)^4$, simplifying the terms. [4]

(ii) Hence show that $(2x + 5)^4 - (2x - 5)^4$ can be written as

$$320x^3 + kx,$$

where the value of the constant k is to be stated. [2]

(iii) Verify that $x = 2$ is a root of the equation

$$(2x + 5)^4 - (2x - 5)^4 = 3680x - 800,$$

and find the other possible values of x. [6]

1 Find and simplify the first three terms in the expansion of $(2 - 3x)^6$ in ascending powers of x. [4]

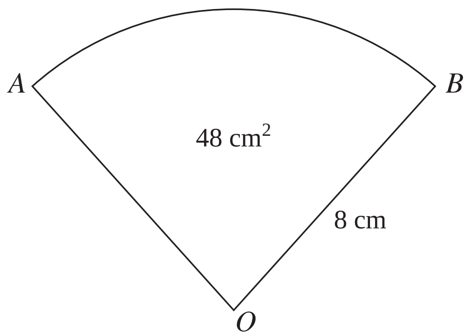
2 A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 3 \quad \text{and} \quad u_{n+1} = 1 - \frac{1}{u_n} \text{ for } n \geqslant 1.$$

(i) Write down the values of u_2, u_3 and u_4. [3]

(ii) Describe the behaviour of the sequence. [1]

3



The diagram shows a sector AOB of a circle with centre O and radius 8 cm. The area of the sector is 48 cm^2.

(i) Find angle AOB, giving your answer in radians. [2]

(ii) Find the area of the segment bounded by the arc AB and the chord AB. [3]

4 The cubic polynomial $ax^3 - 4x^2 - 7ax + 12$ is denoted by $f(x)$.

(i) Given that $(x - 3)$ is a factor of $f(x)$, find the value of the constant a. [3]

(ii) Using this value of a, find the remainder when $f(x)$ is divided by $(x + 2)$. [2]

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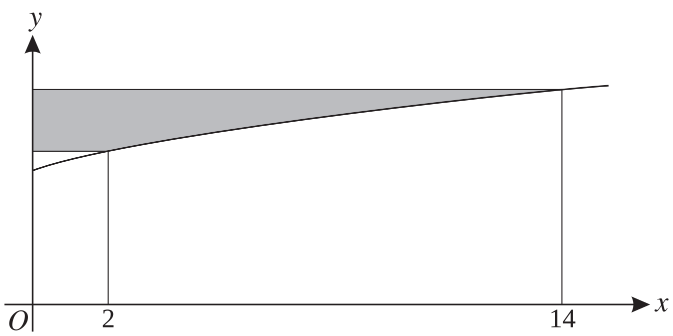
5



The diagram shows the curve $y = 3 + \sqrt{x+2}$.

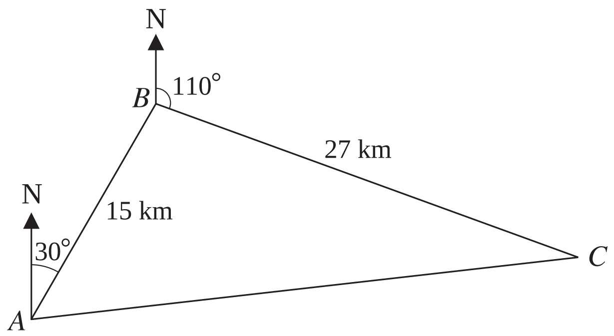
The shaded region is bounded by the curve, the y-axis, and two lines parallel to the x-axis which meet the curve where $x = 2$ and $x = 14$.

(i) Show that the area of the shaded region is given by

$$\int_5^7 (y^2 - 6y + 7)\,dy. \qquad [3]$$

(ii) Hence find the exact area of the shaded region. [4]

6



In the diagram, a lifeboat station is at point A. A distress call is received and the lifeboat travels 15 km on a bearing of 030° to point B. A second call is received and the lifeboat then travels 27 km on a bearing of 110° to arrive at point C. The lifeboat then travels back to the station at A.

(i) Show that angle ABC is 100°. [1]

(ii) Find the distance that the lifeboat has to travel to get from C back to A. [2]

(iii) Find the bearing on which the lifeboat has to travel to get from C to A. [4]

7 **(a)** Find $\int x^3(x^2 - x + 5)\,dx$. [4]

(b) **(i)** Find $\int 18x^{-4}\,dx$. [2]

(ii) Hence evaluate $\int_2^{\infty} 18x^{-4}\,dx$. [2]

8 **(i)** Sketch the curve $y = 2 \times 3^x$, stating the coordinates of any intersections with the axes. [3]

(ii) The curve $y = 2 \times 3^x$ intersects the curve $y = 8^x$ at the point P. Show that the x-coordinate of P may be written as

$$\frac{1}{3 - \log_2 3}.$$ [5]

9 **(a)** **(i)** Show that the equation

$$2 \sin x \tan x - 5 = \cos x$$

can be expressed in the form

$$3 \cos^2 x + 5 \cos x - 2 = 0.$$ [3]

(ii) Hence solve the equation

$$2 \sin x \tan x - 5 = \cos x,$$

giving all values of x, in radians, for $0 \leqslant x \leqslant 2\pi$. [4]

(b) Use the trapezium rule, with four strips each of width 0.25, to find an approximate value for

$$\int_0^1 \cos x \, dx,$$

where x is in radians. Give your answer correct to 3 significant figures. [4]

10 Jamie is training for a triathlon, which involves swimming, running and cycling.

- On Day 1, he swims 2 km and then swims the same distance on each subsequent day.
- On Day 1, he runs 2 km and, on each subsequent day, he runs 0.5 km further than on the previous day. (Thus he runs 2.5 km on Day 2, 3 km on Day 3, and so on.)
- On Day 1 he cycles 2 km and, on each subsequent day, he cycles a distance 10% further than on the previous day.

(i) Find how far Jamie runs on Day 15. [2]

(ii) Verify that the distance cycled in a day first exceeds 12 km on Day 20. [3]

(iii) Find the day on which the total distance cycled, up to and including that day, first exceeds 200 km. [4]

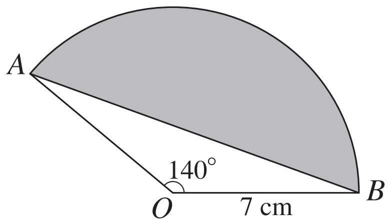
(iv) Find the total distance travelled, by swimming, running and cycling, up to and including Day 30. [4]

1 Find

(i) $\int (x^3 + 8x - 5)\,dx$, **[3]**

(ii) $\int 12\sqrt{x}\,dx$. **[3]**

2



The diagram shows a sector OAB of a circle, centre O and radius 7 cm. The angle AOB is 140°.

(i) Express 140° in radians, giving your answer in an exact form as simply as possible. **[2]**

(ii) Find the perimeter of the segment shaded in the diagram, giving your answer correct to 3 significant figures. **[4]**

3 A sequence of terms $u_1, u_2, u_3, \ldots$ is defined by

$$u_n = 24 - \tfrac{2}{3}n.$$

(i) Write down the exact values of u_1, u_2 and u_3. **[2]**

(ii) Find the value of k such that $u_k = 0$. **[2]**

(iii) Find $\sum_{n=1}^{20} u_n$. **[3]**

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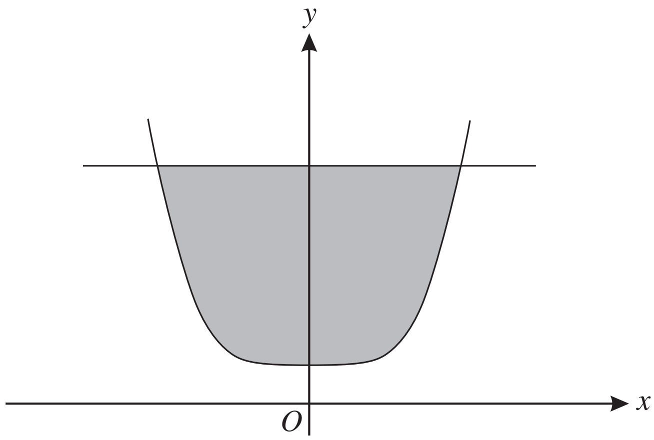
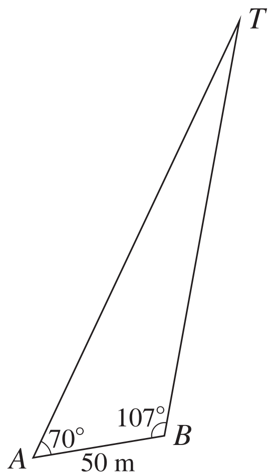
4



The diagram shows the curve $y = x^4 + 3$ and the line $y = 19$ which intersect at $(-2, 19)$ and $(2, 19)$. Use integration to find the exact area of the shaded region enclosed by the curve and the line. **[7]**

5



Some walkers see a tower, T, in the distance and want to know how far away it is. They take a bearing from a point A and then walk for 50 m in a straight line before taking another bearing from a point B. They find that angle TAB is 70° and angle TBA is 107° (see diagram).

(i) Find the distance of the tower from A. **[2]**

(ii) They continue walking in the same direction for another 100 m to a point C, so that AC is 150 m. What is the distance of the tower from C? **[3]**

(iii) Find the shortest distance of the walkers from the tower as they walk from A to C. **[2]**

6 A geometric progression has first term 20 and common ratio 0.9.

(i) Find the sum to infinity. **[2]**

(ii) Find the sum of the first 30 terms. **[2]**

(iii) Use logarithms to find the smallest value of p such that the pth term is less than 0.4. **[4]**

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7 In the binomial expansion of $(k + ax)^4$ the coefficient of x^2 is 24.

(i) Given that a and k are both positive, show that $ak = 2$. **[3]**

(ii) Given also that the coefficient of x in the expansion is 128, find the values of a and k. **[4]**

(iii) Hence find the coefficient of x^3 in the expansion. **[2]**

8 **(a)** Given that $\log_a x = p$ and $\log_a y = q$, express the following in terms of p and q.

(i) $\log_a(xy)$ **[1]**

(ii) $\log_a\left(\frac{a^2x^3}{y}\right)$ **[3]**

(b) **(i)** Express $\log_{10}(x^2 - 10) - \log_{10} x$ as a single logarithm. **[1]**

(ii) Hence solve the equation $\log_{10}(x^2 - 10) - \log_{10} x = 2\log_{10} 3$. **[5]**

9 **(i)** The polynomial $f(x)$ is defined by

$$f(x) = x^3 - x^2 - 3x + 3.$$

Show that $x = 1$ is a root of the equation $f(x) = 0$, and hence find the other two roots. **[6]**

(ii) Hence solve the equation

$$\tan^3 x - \tan^2 x - 3\tan x + 3 = 0$$

for $0 \leqslant x \leqslant 2\pi$. Give each solution for x in an exact form. **[6]**

1 The lengths of the three sides of a triangle are 6.4 cm, 7.0 cm and 11.3 cm.

(i) Find the largest angle in the triangle. **[3]**

(ii) Find the area of the triangle. **[2]**

2 The tenth term of an arithmetic progression is equal to twice the fourth term. The twentieth term of the progression is 44.

(i) Find the first term and the common difference. **[4]**

(ii) Find the sum of the first 50 terms. **[2]**

3 Use logarithms to solve the equation $7^x = 2^{x+1}$, giving the value of x correct to 3 significant figures. **[5]**

4 **(i)** Find the binomial expansion of $(x^2 - 5)^3$, simplifying the terms. **[4]**

(ii) Hence find $\int (x^2 - 5)^3 \, dx$. **[4]**

5 Solve each of the following equations for $0° \leqslant x \leqslant 180°$.

(i) $\sin 2x = 0.5$ **[3]**

(ii) $2\sin^2 x = 2 - \sqrt{3}\cos x$ **[5]**

6 The gradient of a curve is given by $\frac{dy}{dx} = 3x^2 + a$, where a is a constant. The curve passes through the points $(-1, 2)$ and $(2, 17)$. Find the equation of the curve. **[8]**

7 The polynomial $f(x)$ is given by $f(x) = 2x^3 + 9x^2 + 11x - 8$.

(i) Find the remainder when $f(x)$ is divided by $(x + 2)$. **[2]**

(ii) Use the factor theorem to show that $(2x - 1)$ is a factor of $f(x)$. **[2]**

(iii) Express $f(x)$ as a product of a linear factor and a quadratic factor. **[3]**

(iv) State the number of real roots of the equation $f(x) = 0$, giving a reason for your answer. **[2]**

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8

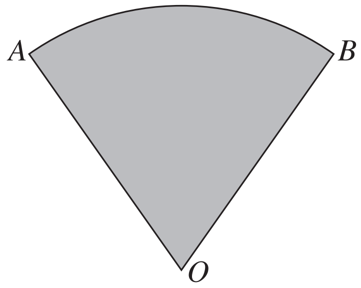

Fig. 1

Fig. 1 shows a sector AOB of a circle, centre O and radius OA. The angle AOB is 1.2 radians and the area of the sector is $60\,\text{cm}^2$.

(i) Find the perimeter of the sector. **[4]**

A pattern on a T-shirt, the start of which is shown in Fig. 2, consists of a sequence of similar sectors. The first sector in the pattern is sector AOB from Fig. 1, and the area of each successive sector is $\frac{3}{5}$ of the area of the previous one.

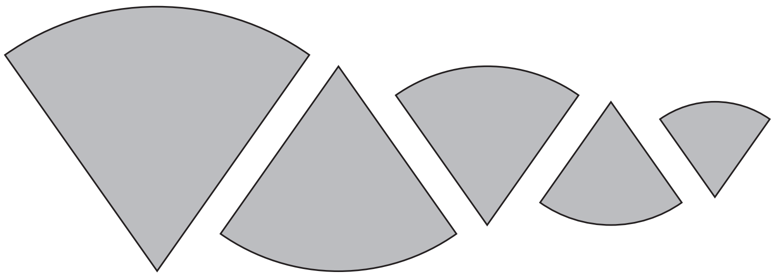

Fig. 2

(ii) **(a)** Find the area of the fifth sector in the pattern. **[2]**

(b) Find the total area of the first ten sectors in the pattern. **[2]**

(c) Explain why the total area will never exceed a certain limit, no matter how many sectors are used, and state the value of this limit. **[3]**

9 **(i)** Sketch the graph of $y = 4k^x$, where k is a constant such that $k > 1$. State the coordinates of any points of intersection with the axes. **[2]**

(ii) The point P on the curve $y = 4k^x$ has its y-coordinate equal to $20k^2$. Show that the x-coordinate of P may be written as $2 + \log_k 5$. **[4]**

(iii) **(a)** Use the trapezium rule, with two strips each of width $\frac{1}{2}$, to find an expression for the approximate value of

$$\int_0^1 4k^x \, dx.$$ **[3]**

(b) Given that this approximate value is equal to 16, find the value of k. **[3]**