The Grange School
Maths Department

Decision 1
OCR Past Papers

1 An airline allows each passenger to carry a maximum of 25 kg in luggage. The four members of the Adams family have bags of the following weights (to the nearest kg):

Mr Adams:	10	4	2			
Mrs Adams:	13	3	7	5	2	4
Sarah Adams:	5	8	2	5		
Tim Adams:	10	5	3	5	3	

The bags need to be grouped into bundles of 25 kg maximum so that each member of the family can carry a bundle of bags.

(i) Use the first-fit method to group the bags into bundles of 25 kg maximum. Start with the bags belonging to Mr Adams, then those of Mrs Adams, followed by Sarah and finally Tim. [3]

(ii) Use the first-fit decreasing method to group the same bags into bundles of 25 kg maximum. [3]

(iii) Suggest a reason why the grouping of the bags in part **(i)** might be easier for the family to carry. [1]

2 A baker can make apple cakes, banana cakes and cherry cakes.

The baker has exactly enough flour to make either 30 apple cakes or 20 banana cakes or 40 cherry cakes.

The baker has exactly enough sugar to make either 30 apple cakes or 40 banana cakes or 30 cherry cakes.

The baker has enough apples for 20 apple cakes, enough bananas for 25 banana cakes and enough cherries for 10 cherry cakes.

The baker has an order for 30 cakes.

The profit on each apple cake is 4p, on each banana cake is 3p and on each cherry cake is 2p. The baker wants to maximise the profit on the order.

(i) The availability of flour leads to the constraint $4a + 6b + 3c \leqslant 120$. Give the meaning of each of the variables a, b and c in this constraint. [2]

(ii) Use the availability of sugar to give a second constraint of the form $Xa + Yb + Zc \leqslant 120$, where X, Y and Z are numbers to be found. [2]

(iii) Write down a constraint from the total order size. Write down constraints from the availability of apples, bananas and cherries. [3]

(iv) Write down the objective function to be maximised. [1]

[You are **not** required to solve the resulting LP problem.]

3 A *simple* graph is one in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself.

A *connected* graph is one in which every vertex is connected, directly or indirectly, to every other vertex.

A *simply connected* graph is one that is both simple and connected.

(i) A simply connected graph is drawn with 6 vertices and 9 arcs.

(a) What is the sum of the orders of the vertices? [1]

(b) Explain why if the graph has two vertices of order 5 it cannot have any vertices of order 1. [2]

(c) Explain why the graph cannot have three vertices of order 5. [2]

(ii) Draw an example of a simply connected graph with 6 vertices and 9 arcs in which one of the vertices has order 5 and all the orders of the vertices are odd numbers. [2]

(iii) Draw an example of a simply connected graph with 6 vertices and 9 arcs that is also Eulerian. [2]

Jan 2007

4 Answer this question on the insert provided.

The table shows the distances, in units of 100 m, between seven houses, A to G.

	A	B	C	D	E	F	G
A	0	4	5	3	2	5	6
B	4	0	1	2	4	7	6
C	5	1	0	3	4	6	7
D	3	2	3	0	2	6	4
E	2	4	4	2	0	6	6
F	5	7	6	6	6	0	10
G	6	6	7	4	6	10	0

(i) Use Prim's algorithm on the table in the insert to find a minimum spanning tree. Start by crossing out row A. Show which entries in the table are chosen and indicate the order in which the rows are deleted. Draw your minimum spanning tree and state its total weight. [6]

Harry is an estate agent. He must visit each of the houses A to G to photograph them. The distances, in units of 100 m, from Harry's office (H) to each of the houses are listed below.

House	A	B	C	D	E	F	G
Distance from H	12	14	15	15	13	16	16

Harry wants to find the shortest route that starts at his office and visits each of the houses before returning to his office.

(ii) Which standard network problem does Harry need to solve? [1]

(iii) Use your answer from part **(i)** to calculate a lower bound for the length of Harry's route, showing all your working. [3]

(iv) Use the nearest neighbour method, starting from Harry's office, to find a tour that visits each of the houses. Hence find an upper bound for the length of Harry's route. [4]

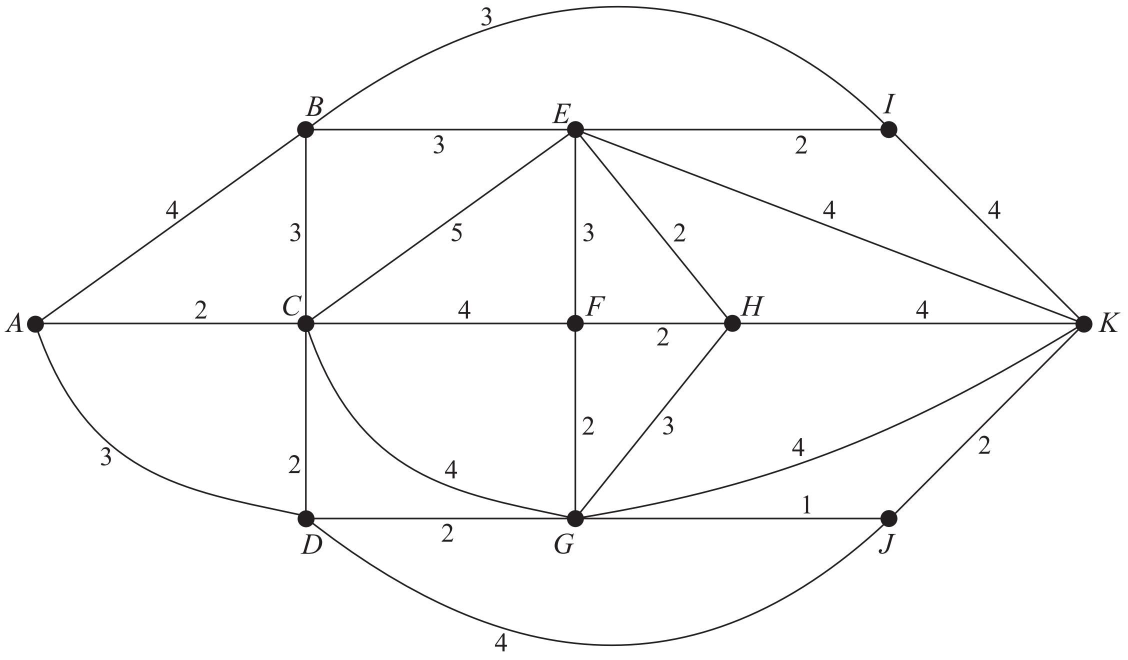
Jan 2007

5 **Answer part (i) of this question on the insert provided.**

Rhoda Raygh enjoys driving but gets extremely irritated by speed cameras.

The network represents a simplified map on which the arcs represent roads and the weights on the arcs represent the numbers of speed cameras on the roads.

The sum of the weights on the arcs is 72.



(i) Rhoda lives at Ayton (A) and works at Kayton (K). Use Dijkstra's algorithm on the diagram in the insert to find the route from A to K that involves the least number of speed cameras and state the number of speed cameras on this route. [7]

(ii) In her job Rhoda has to drive along each of the roads represented on the network to check for overhanging trees. This requires finding a route that covers every arc at least once, starting and ending at Kayton (K). Showing all your working, find a suitable route for Rhoda that involves the least number of speed cameras and state the number of speed cameras on this route. [6]

(iii) If Rhoda checks the roads for overhanging trees on her way home, she will instead need a route that covers every arc at least once, starting at Kayton and ending at Ayton. Calculate the least number of speed cameras on such a route, explaining your reasoning. [3]

6 Consider the linear programming problem:

$$\begin{array}{ll} \text{maximise} & P = 3x - 5y + 4z, \\ \text{subject to} & x + 2y - 3z \leqslant 12, \\ & 2x + 5y - 8z \leqslant 40, \\ \text{and} & x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0. \end{array}$$

(i) Represent the problem as an initial Simplex tableau. [3]

(ii) Explain why it is not possible to pivot on the z column of this tableau. Identify the entry on which to pivot for the first iteration of the Simplex algorithm. Explain how you made your choice of column and row. [3]

(iii) Perform **one** iteration of the Simplex algorithm. Write down the values of x, y and z after this iteration. [3]

(iv) Explain why P has no finite maximum. [1]

The coefficient of z in the objective is changed from +4 to −40.

(v) Describe the changes that this will cause to the initial Simplex tableau and the tableau that results after one iteration. What is the maximum value of P in this case? [4]

Now consider this linear programming problem:

$$\begin{array}{ll} \text{maximise} & Q = 3x - 5y + 7z, \\ \text{subject to} & x + 2y - 3z \leqslant 12, \\ & 2x - 7y + 10z \leqslant 40, \\ \text{and} & x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0. \end{array}$$

Do **not** use the Simplex algorithm for these parts.

(vi) By adding the two constraints, show that Q has a finite maximum. [1]

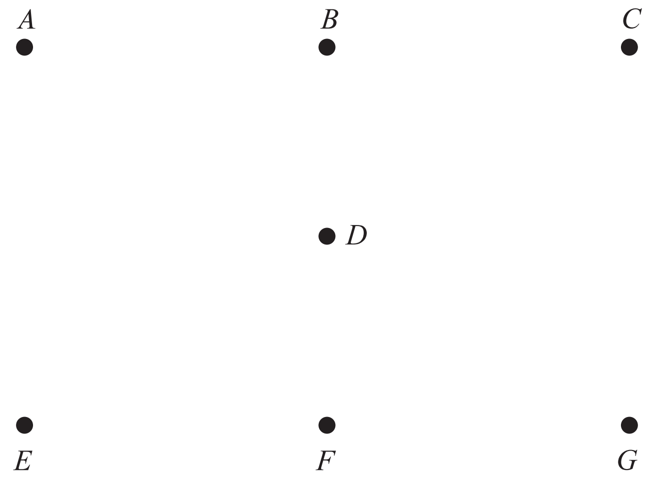
(vii) There is an optimal point with $y = 0$. By substituting $y = 0$ in the two constraints, calculate the values of x and z that maximise Q when $y = 0$. [3]

4 **(i)**

	↓						
	A	*B*	*C*	*D*	*E*	*F*	*G*
A	0	4	5	3	2	5	6
B	4	0	1	2	4	7	6
C	5	1	0	3	4	6	7
D	3	2	3	0	2	6	4
E	2	4	4	2	0	6	6
F	5	7	6	6	6	0	10
G	6	6	7	4	6	10	0

Order in which rows were deleted: *A* ....

Minimum spanning tree:



Total weight: ....

(ii) ....

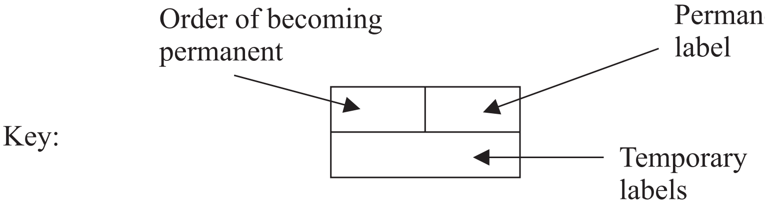
(iii) ....

Lower bound: ....

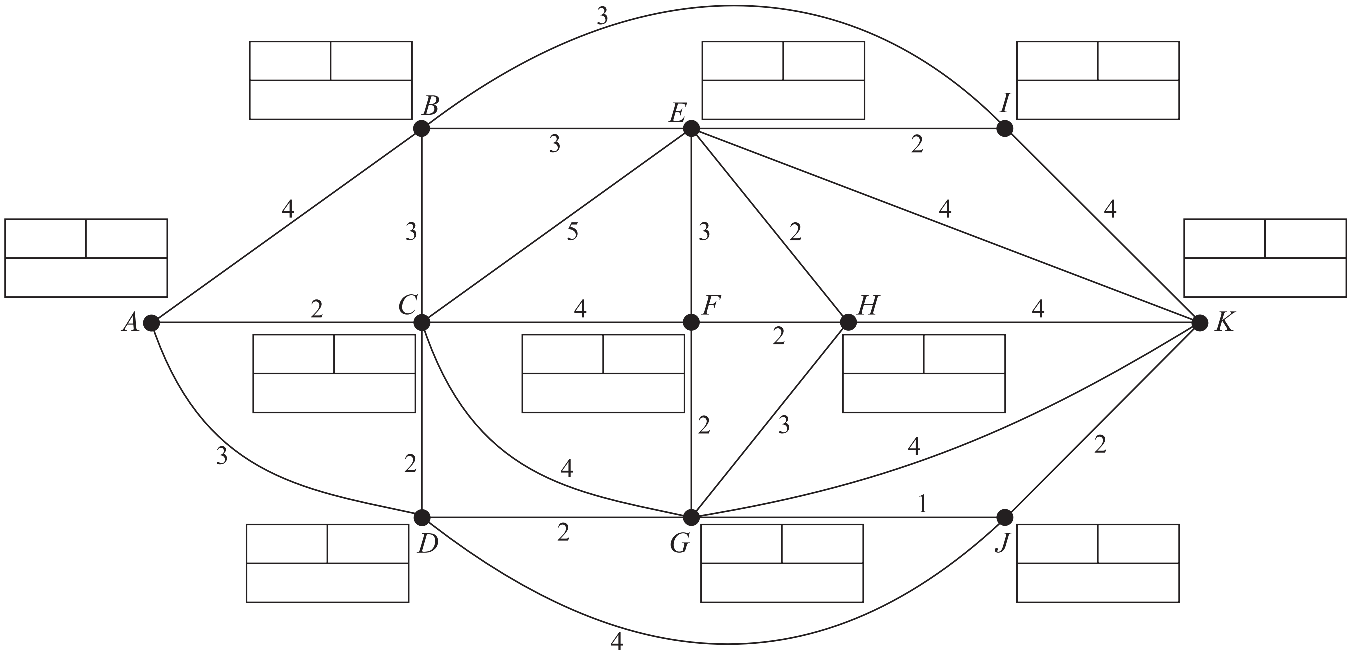
(iv) Tour: ....

Upper bound: ....

5 **(i)**



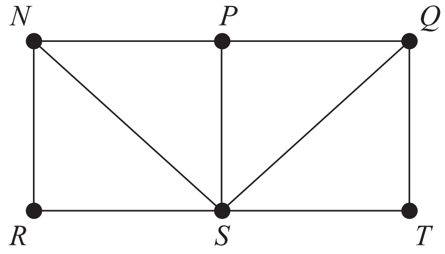
Do not cross out your working values (temporary labels)



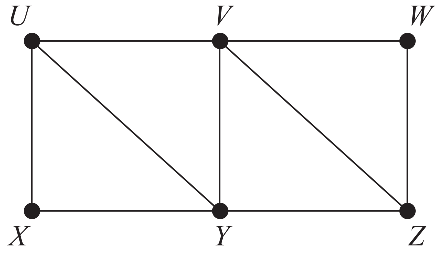
Route: ....

Number of speed cameras on route: ....

1 Two graphs A and B are shown below.



Graph A



Graph B

(i) Write down an example of a cycle on graph A. [1]

(ii) Why is $U–Y–V–Z–Y–X$ not a path on graph B? [1]

(iii) How many arcs would there be in a spanning tree for graph A? [1]

(iv) For each graph state whether it is Eulerian, semi-Eulerian or neither. [2]

(v) The graphs show designs to be etched on metal plates. The etching tool is positioned at a starting point and follows a route without repeating any arcs. It may be lifted off and positioned at a new starting point. What is the smallest number of times that the etching tool must be positioned, including the initial position, to draw each graph? [2]

An arc is drawn connecting Q to U, so that the two graphs become one. The resulting graph is not Eulerian.

(vi) Extra arcs are then added to make an Eulerian graph. What is the smallest number of extra arcs that need to be added? [2]

2 A landscape gardener is designing a garden. Part of the garden will be decking, part will be flowers and the rest will be grass. Let d be the area of decking, f be the area of flowers and g be the area of grass, all measured in m^2.

The total area of the garden is $120\,\text{m}^2$ of which at least $40\,\text{m}^2$ must be grass. The area of decking must not be greater than the area of flowers. Also, the area of grass must not be more than four times the area of decking.

Each square metre of grass will cost £5, each square metre of decking will cost £10 and each square metre of flowers will cost £20. These costs include labour. The landscape gardener has been instructed to come up with the design that will cost the least.

(i) Write down a constraint in d, f and g from the total area of the garden. [1]

(ii) Explain why the constraint $g \leqslant 4d$ is required. [1]

(iii) Write down a constraint from the requirement that the area of decking must not be greater than the area of flowers. [1]

(iv) Write down a constraint from the requirement that at least $40\,\text{m}^2$ of the garden must be grass and write down the minimum feasible values for each of d and f. [3]

(v) Write down the objective function to be minimised. [1]

(vi) Write down the resulting LP problem, using slack variables to express the constraints from parts **(ii)** and **(iii)** as equations. [3]

(You are **not** required to solve the resulting LP problem.)

3 **(i)** Use shuttle sort to sort the five numbers 8, 6, 9, 7, 5 into increasing order. Write down the list that results at the end of each pass. Calculate and record the number of comparisons and the number of swaps that are made in each pass. [6]

(ii) The algorithm below is **part** of another method for sorting a list into increasing order. Apply it to the list 8, 6, 9, 7, 5. Show the result of each step. [5]

```
Step 1: Input the original list and call it list A.
Step 2: Remove the first item in list A and call this item X.
Step 3: If the first item remaining in list A is less than X move it to list B,
        otherwise move it to list C.
Step 4: If the next item remaining in list A is less than X move it to become
        the next item in list B, otherwise move it to become the next item
        in list C.
Step 5: If there are still items in list A, repeat Step 4.
Step 6: Count the number of items in list B and call this N.
Step 7: Put the items in list B at positions 1 to N of list A, item X at position
        N + 1 of list A and the items in list C at positions N + 2 onwards of
        list A.
Step 8: Display list A.
```

June 2007

4 Consider the linear programming problem:

$$\begin{array}{ll} \text{maximise} & P = 3x - 5y, \\ \text{subject to} & x + 5y \leqslant 12, \\ & x - 5y \leqslant 10, \\ & 3x + 10y \leqslant 45, \\ \text{and} & x \geqslant 0,\ y \geqslant 0. \end{array}$$

(i) Represent the problem as an initial Simplex tableau. [3]

(ii) Identify the entry on which to pivot for the first iteration of the Simplex algorithm. Explain how you made your choice of column and row. [2]

(iii) Perform **one** iteration of the Simplex algorithm. Write down the values of x, y and P after this iteration. [6]

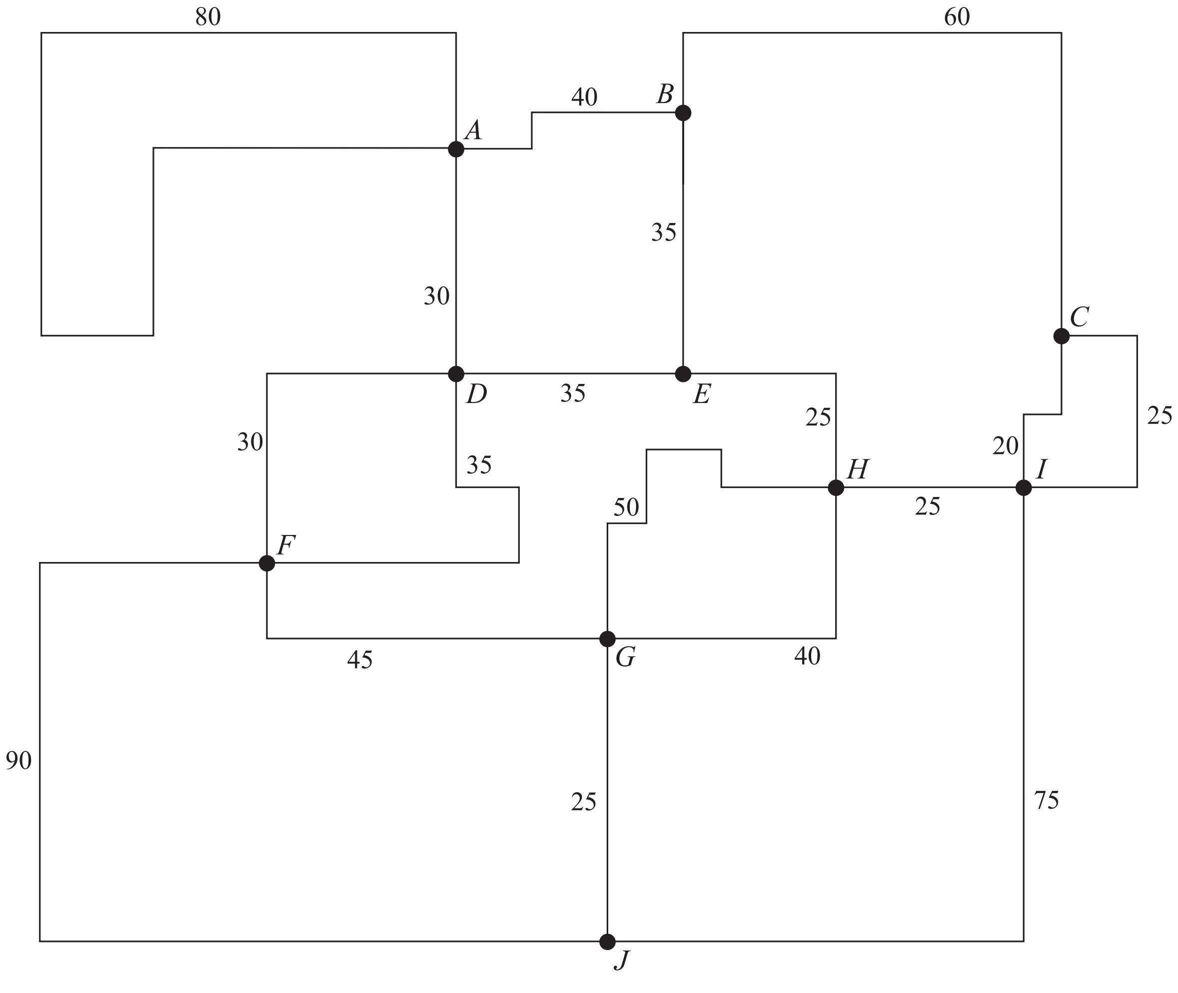
(iv) Show that $x = 11$, $y = 0.2$ is a feasible solution and that it gives a bigger value of P than that in part **(iii)**. [2]

June 2007

5 Answer this question on the insert provided.

The network below represents a simplified map of a building. The arcs represent corridors and the weights on the arcs represent the lengths of the corridors, in metres.

The sum of the weights on the arcs is 765 metres.



(i) Janice is the cleaning supervisor in the building. She is at the position marked as J when she is called to attend a cleaning emergency at B. On the network in the insert, use Dijkstra's algorithm, starting from vertex J and continuing until B is given a permanent label, to find the shortest path from J to B and the length of this path. [7]

(ii) In her job Janice has to walk along each of the corridors represented on the network. This requires finding a route that covers every arc at least once, starting and ending at J. Showing all your working, find the shortest distance that Janice must walk to check all the corridors. [5]

The labelled vertices represent 'cleaning stations'. Janice wants to visit every cleaning station using the shortest possible route. She produces a simplified network with no repeated arcs and no arc that joins a vertex to itself.

(iii) On the insert, complete Janice's simplified network. Which standard network problem does Janice need to solve to find the shortest distance that she must travel? [4]

June 2007

6 Answer this question on the insert provided.

The table shows the distances, in miles, along the direct roads between six villages, *A* to *F*. A dash (–) indicates that there is no direct road linking the villages.

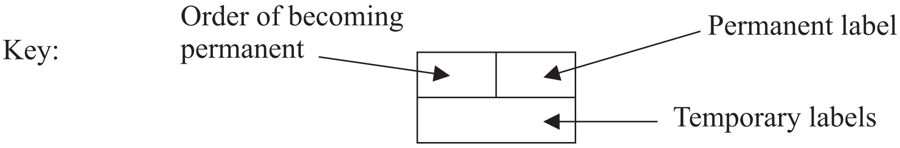
	A	*B*	*C*	*D*	*E*	*F*
A	–	6	3	–	–	–
B	6	–	5	6	–	14
C	3	5	–	8	4	10
D	–	6	8	–	3	8
E	–	–	4	3	–	–
F	–	14	10	8	–	–

(i) On the table in the insert, use Prim's algorithm to find a minimum spanning tree. Start by crossing out row *A*. Show which entries in the table are chosen and indicate the order in which the rows are deleted. Draw your minimum spanning tree and state its total weight. [6]

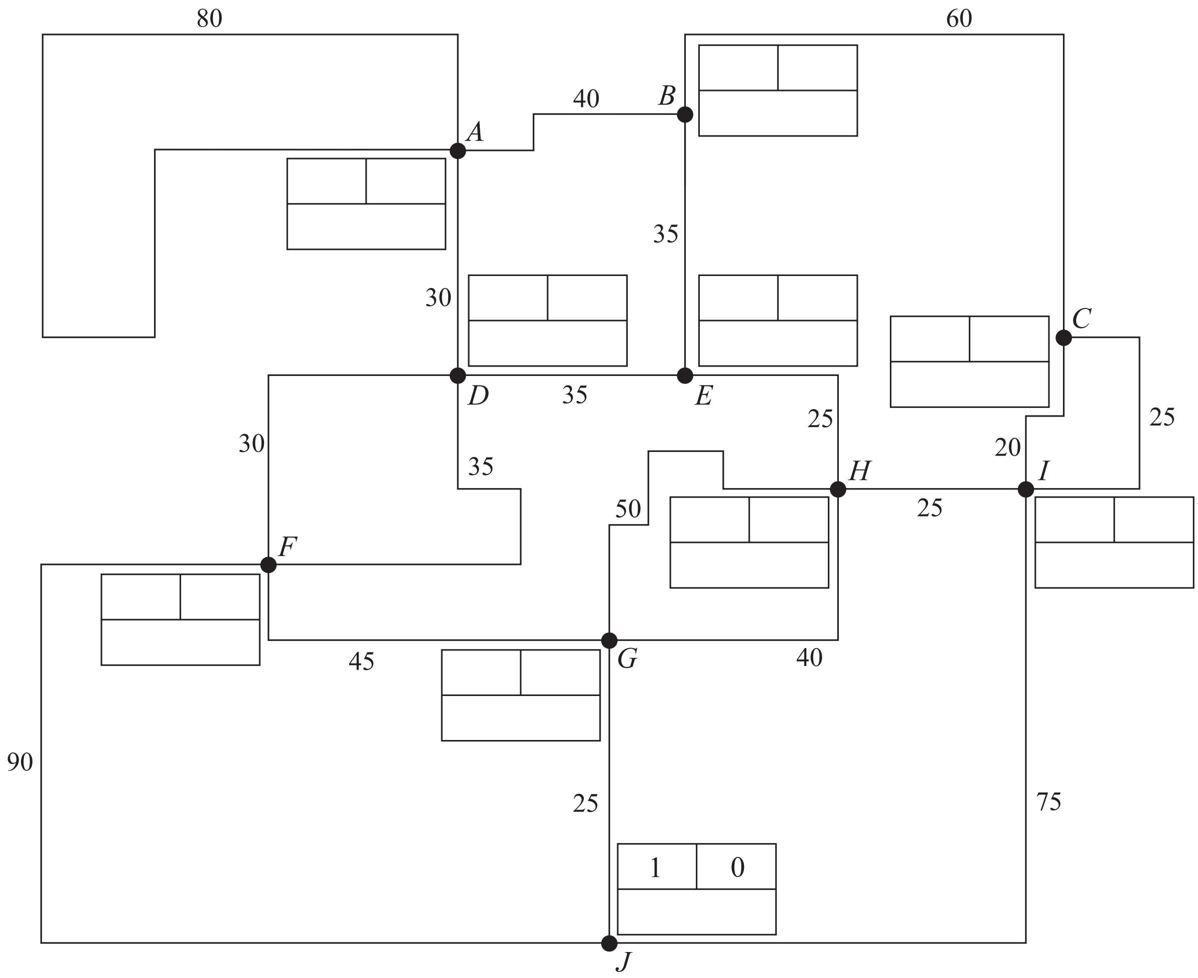
(ii) By deleting vertex *B* and the arcs joined to vertex *B*, calculate a lower bound for the length of the shortest cycle through all the vertices. [3]

(iii) Apply the nearest neighbour method to the table above, starting from *F*, to find a cycle that passes through every vertex and use this to write down an upper bound for the length of the shortest cycle through all the vertices. [4]

5 **(i)**



Do not cross out your working values (temporary labels)



Shortest path from J to B: ....

Length of path: .... metres

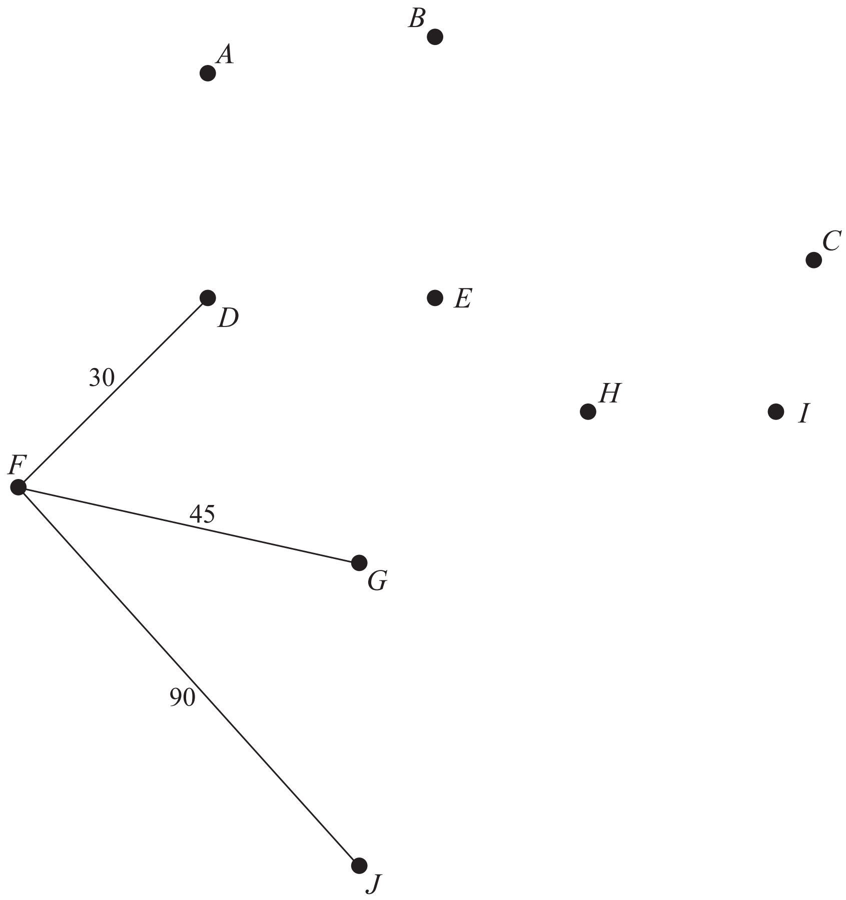
(ii) ....

....

....

Length of shortest route that starts and ends at J and covers every arc = ... metres

(iii)



....

June 2007 Insert

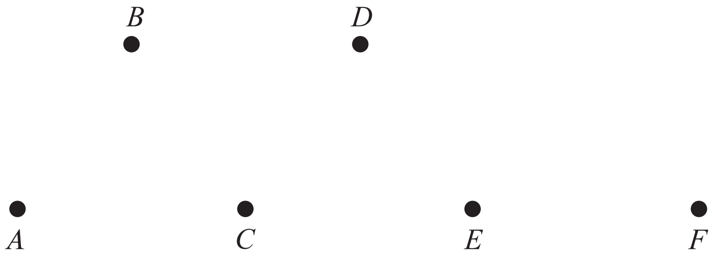
6 **(i)**

↓

	A	*B*	*C*	*D*	*E*	*F*
A	–	6	3	–	–	–
B	6	–	5	6	–	14
C	3	5	–	8	4	10
D	–	6	8	–	3	8
E	–	–	4	3	–	–
F	–	14	10	8	–	–

Order in which rows were deleted: *A*..

Minimum spanning tree:



Total weight: .... miles

(ii) ....

....

Lower bound: .... miles

(iii) Cycle: ....

....

Upper bound: .... miles

1 Five boxes weigh 5 kg, 2 kg, 4 kg, 3 kg and 8 kg. They are stacked, in the order given, with the first box at the top of the stack. The boxes are to be packed into bins that can each hold up to 10 kg.

(i) Use the first-fit method to put the boxes into bins. Show clearly which boxes are packed in which bins. [2]

(ii) Use the first-fit decreasing method to put the boxes into bins. You do not need to use an algorithm for sorting. Show clearly which boxes are packed in which bins. [2]

(iii) Why might the first-fit decreasing method not be practical? [1]

(iv) Show that if the bins can only hold up to 8 kg each it is still possible to pack the boxes into three bins. [1]

2 A puzzle involves a 3 by 3 grid of squares, numbered 1 to 9, as shown in Fig. 1a below. Eight of the squares are covered by blank tiles. Fig. 1b shows the puzzle with all of the squares covered except for square 4. This arrangement of tiles will be called position 4.

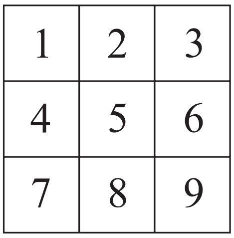


Fig. 1a

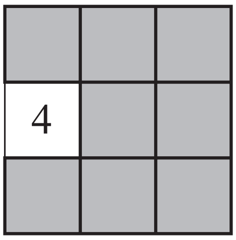


Fig. 1b

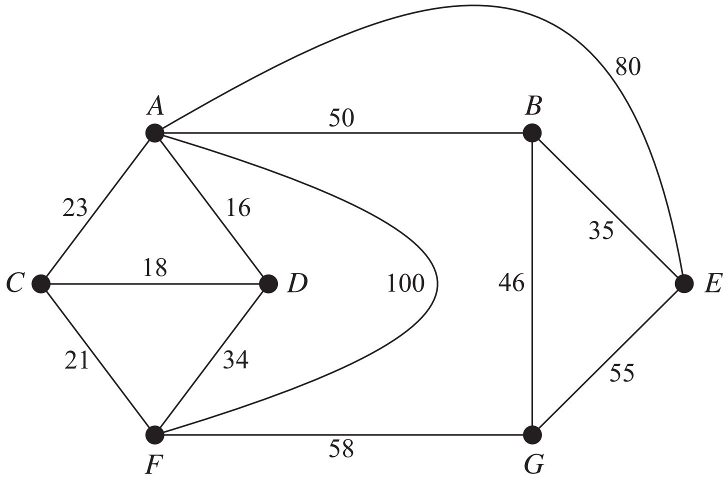
A move consists of sliding a tile into the empty space. From position 4, the next move will result in position 1, position 5 or position 7.

(i) Draw a graph with nine vertices to represent the nine positions and arcs that show which positions can be reached from one another in one move. What is the least number of moves needed to get from position 1 to position 9? [3]

(ii) State whether the graph from part **(i)** is Eulerian, semi-Eulerian or neither. Explain how you know which it is. [2]

Jan 2008

3 Answer this question on the insert provided.



(i) This diagram shows a network. The insert has a copy of this network together with a list of the arcs, sorted into increasing order of weight. Use Kruskal's algorithm on the insert to find a minimum spanning tree for this network. Draw your tree and give its total weight. [5]

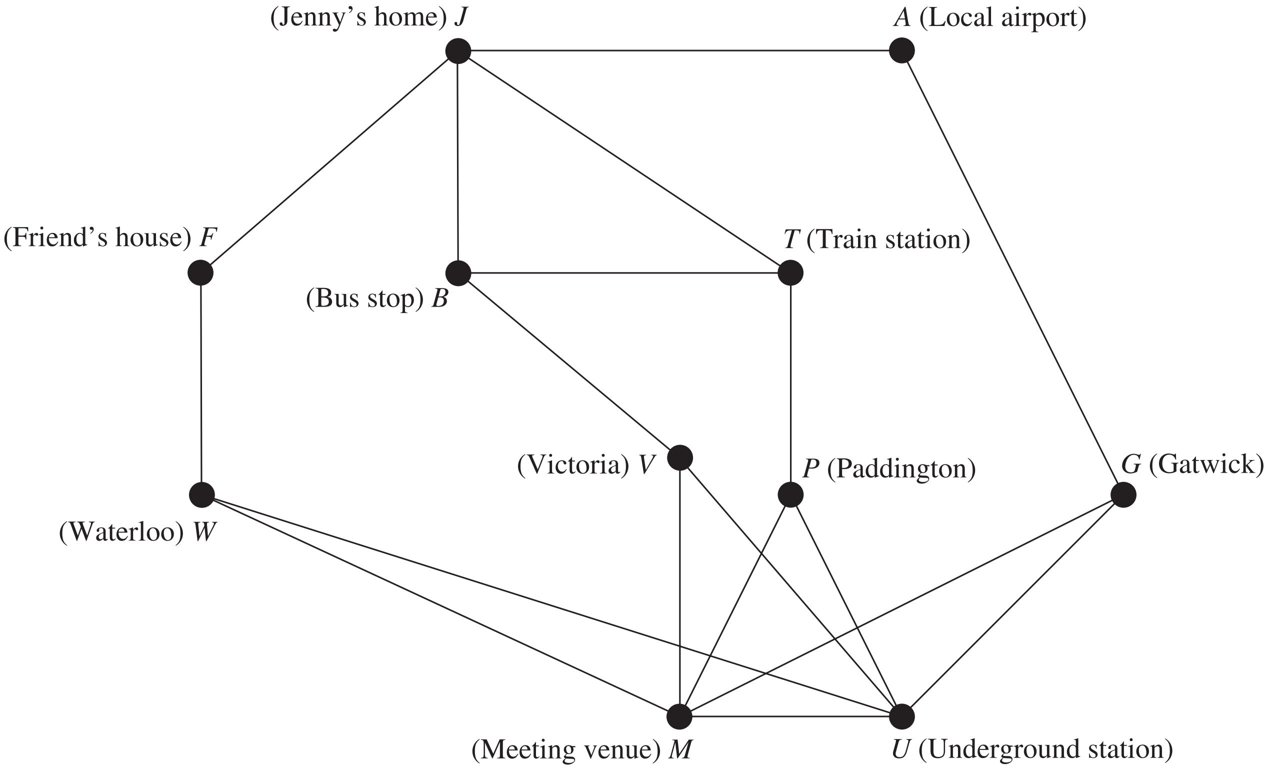
(ii) Use your answer to part **(i)** to find the weight of a minimum spanning tree for the network with vertex *G*, and all the arcs joined to *G*, removed. Hence find a lower bound for the travelling salesperson problem on the original network. [3]

(iii) Apply the nearest neighbour method, starting from vertex *A*, to find an upper bound for the travelling salesperson problem on the original network. [3]

Jan 2008

4 Answer this question on the insert provided.

Jenny needs to travel to London to arrive in time for a morning meeting. The graph below represents the various travel options that are available to her.



It takes Jenny 120 minutes to drive from her home to the local airport and check in (arc JA). The journey from the local airport to Gatwick takes 80 minutes. From Gatwick to the underground station takes 60 minutes, and walking from the underground station to the meeting venue takes 15 minutes. Alternatively, Jenny could get a taxi from Gatwick to the meeting venue; this takes 80 minutes.

It takes Jenny 15 minutes to drive from her house to the train station. Alternatively, she can walk to the bus stop, which takes 5 minutes, and then get a bus to the train station, taking another 20 minutes. From the train station to Paddington takes 300 minutes, and from Paddington to the underground station takes a further 20 minutes. Alternatively, Jenny could walk from Paddington to the meeting venue, taking 30 minutes.

Jenny can catch a coach from her local bus stop to Victoria, taking 400 minutes. From Victoria she can either travel to the underground station, which takes 10 minutes, or she can walk to the meeting venue, which takes 15 minutes.

The final option available to Jenny is to drive to a friend's house, taking 240 minutes, and then continue the journey into London by train. The journey from her friend's house to Waterloo takes Jenny 30 minutes. From here she can either go to the underground station, which takes 20 minutes, or walk to the meeting venue, which takes 40 minutes.

(i) Weight the arcs on the graph in the insert to show these times. Apply Dijkstra's algorithm, starting from J, to give a permanent label and order of becoming permanent at each vertex. Stop when you have assigned a permanent label to vertex M. Write down the route of the shortest path from J to M. [9]

(ii) What does the value of the permanent label at M represent? [1]

(iii) Give two reasons why Jenny might choose to use a different route from J to M. [2]

5 Mark wants to decorate the walls of his study. The total wall area is 24 m^2. Mark can cover the walls using any combination of three materials: panelling, paint and pinboard. He wants at least 2 m^2 of pinboard and at least 10 m^2 of panelling.

Panelling costs £8 per m^2 and it will take Mark 15 minutes to put up 1 m^2 of panelling. Paint costs £4 per m^2 and it will take Mark 30 minutes to paint 1 m^2. Pinboard costs £10 per m^2 and it will take Mark 20 minutes to put up 1 m^2 of pinboard. He has all the equipment that he will need for the decorating jobs.

Mark is able to spend up to £150 on the materials for the decorating. He wants to know what area should be covered with each material to enable him to complete the whole job in the shortest time possible.

Mark models the problem as an LP with five constraints. His constraints are:

$$x + y + z = 24,$$
$$4x + 2y + 5z \leqslant 75,$$
$$x \geqslant 10,$$
$$y \geqslant 0,$$
$$z \geqslant 2.$$

(i) Identify the meaning of each of the variables x, y and z. [2]

(ii) Show how the constraint $4x + 2y + 5z \leqslant 75$ was formed. [2]

(iii) Write down an objective function, to be minimised. [1]

Mark rewrites the first constraint as $z = 24 - x - y$ and uses this to eliminate z from the problem.

(iv) Rewrite and simplify the objective and the remaining four constraints as functions of x and y only. [3]

(v) Represent your constraints from part **(iv)** graphically and identify the feasible region. Your graph should show x and y values from 9 to 15 only. [4]

6 **(i)** Represent the linear programming problem below by an initial Simplex tableau. [2]

Maximise $P = 25x + 14y - 32z$,

subject to $6x - 4y + 3z \leqslant 24$,
$5x - 3y + 10z \leqslant 15$,

and $x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$.

(ii) Explain how you know that the first iteration will use a pivot from the x column. Show the calculations used to find the pivot element. [3]

(iii) Perform **one** iteration of the Simplex algorithm. Show how each row was calculated and write down the values of x, y, z and P that result from this iteration. [7]

(iv) Explain why the Simplex algorithm cannot be used to find the optimal value of P for this problem. [1]

7 In this question, the function INT(X) is the largest integer less than or equal to X. For example,

$$\begin{aligned}&\text{INT}(3.6) = 3,\\&\text{INT}(3) = 3,\\&\text{INT}(-3.6) = -4.\end{aligned}$$

Consider the following algorithm.

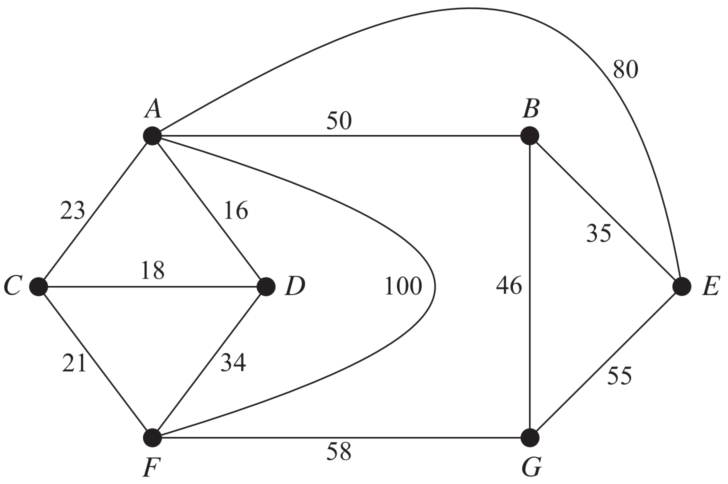
```
Step 1    Input $B$
Step 2    Input $N$
Step 3    Calculate $F = N \div B$
Step 4    Let $G = \text{INT}(F)$
Step 5    Calculate $H = B \times G$
Step 6    Calculate $C = N - H$
Step 7    Output $C$
Step 8    Replace $N$ by the value of $G$
Step 9    If $N = 0$ then stop, otherwise go back to Step 3
```

(i) Apply the algorithm with the inputs $B = 2$ and $N = 5$. Record the values of F, G, H, C and N each time Step 9 is reached. [5]

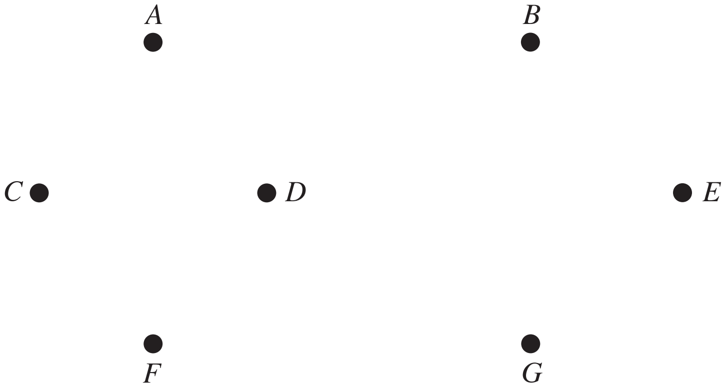
(ii) Explain what happens when the algorithm is applied with the inputs $B = 2$ and $N = -5$. [4]

(iii) Apply the algorithm with the inputs $B = 10$ and $N = 37$. Record the values of F, G, H, C and N each time Step 9 is reached. What are the output values when $B = 10$ and N is any positive integer? [4]

3 **(i)**



AD = 16
CD = 18
CF = 21
AC = 23
DF = 34
BE = 35
BG = 46
AB = 50
EG = 55
FG = 58
AE = 80
AF = 100



Total weight of arcs in minimum spanning tree = ...

(ii) ....

....

Weight of spanning tree for the network with vertex *G* removed = ...

....

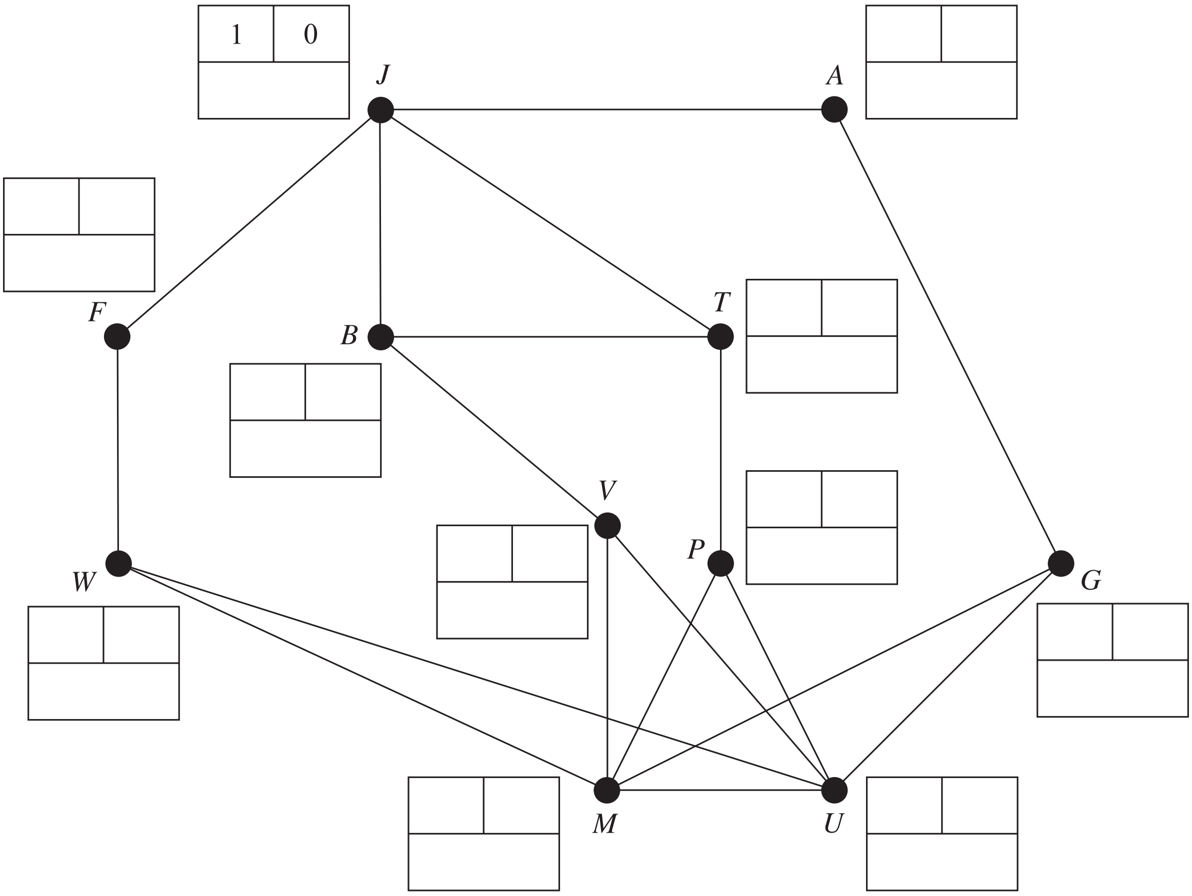
Lower bound for travelling salesperson problem on original network = ...

(iii) ....

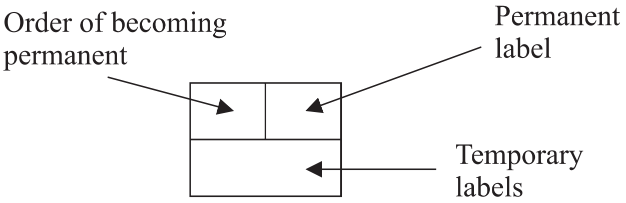
Upper bound for travelling salesperson problem on original network = ...

[Turn over

4 **(i)**



Key:



Do not cross out your working values (temporary labels)

Route of shortest path from J to M: ....

(ii) ....

....

(iii) ....

....

....

1 This question is about using bubble sort to sort a list of numbers into **increasing** order.

(i) Which numbers, if any, can be guaranteed to be in their correct final position after the first pass? [1]

Suppose now that the original, unsorted list was 3, 2, 1, 5, 4.

(ii) Write down the list that results after one pass through bubble sort. How many comparisons and how many swaps were used in this pass? [2]

(iii) Write down the list that results after a second pass through bubble sort. How many more passes will be required until the algorithm terminates? [2]

Bubble sort is a quadratic order algorithm.

(iv) A computer takes 0.2 seconds to sort a list of 500 numbers using bubble sort. Approximately how long will it take to sort a list of 3000 numbers? [2]

2 A *simple* graph is one in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself.

A *connected* graph is one in which every vertex is joined, directly or indirectly, to every other vertex.

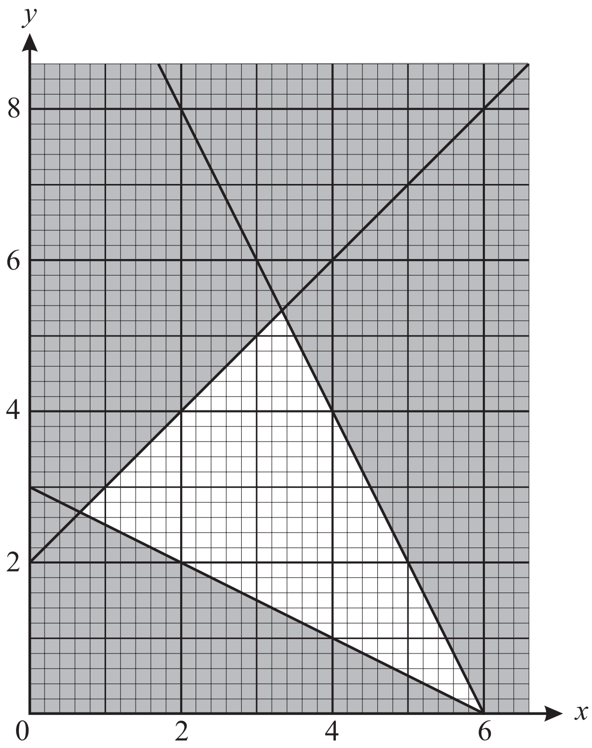
A *simply connected* graph is one that is both simple and connected.

(i) Draw an Eulerian graph with four vertices, of orders 2, 2, 4 and 4, and no others. Explain why your graph is not simply connected. [3]

(ii) Draw a non-Eulerian graph with four vertices, of orders 2, 2, 4 and 4, and no others. Explain why your graph is non-Eulerian even though its vertices are all of even order. [3]

June 2008

3 The constraints of a linear programming problem are represented by the graph below. The feasible region is the unshaded region, including its boundaries.



(i) Write down the inequalities that define the feasible region. [4]

(ii) Calculate the coordinates of the three vertices of the feasible region. [4]

The objective is to maximise $5x + 3y$.

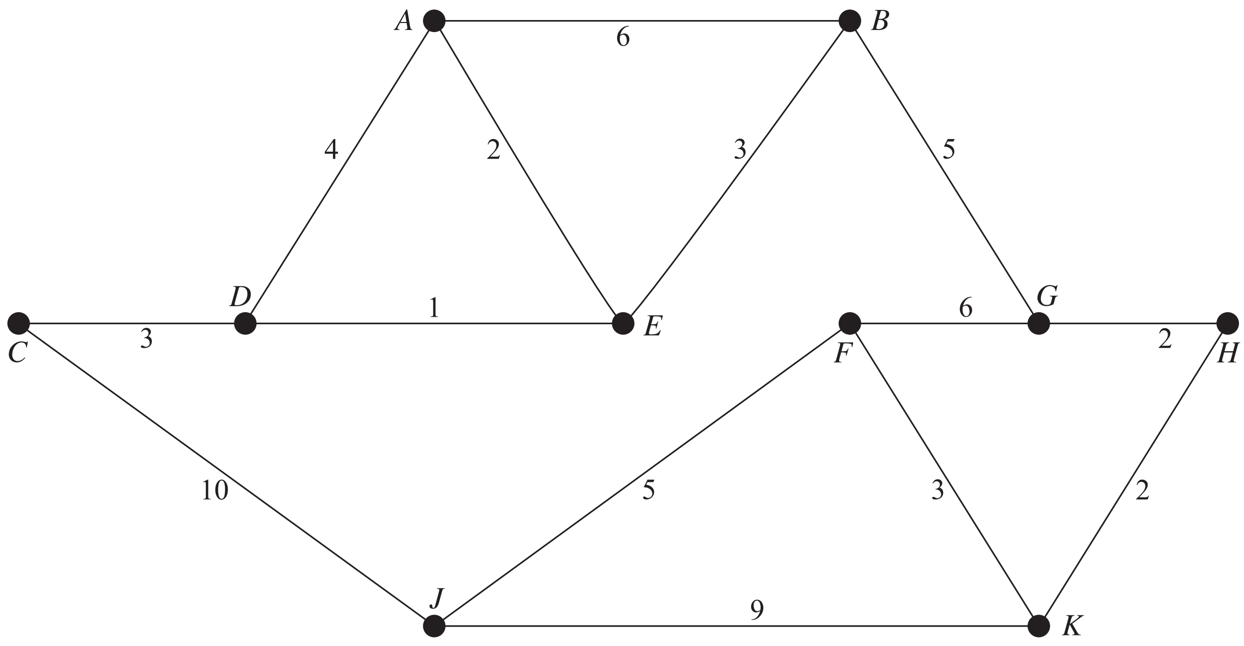
(iii) Find the values of x and y at the optimal point, and the corresponding maximum value of $5x + 3y$. [3]

The objective is changed to maximise $5x + ky$, where k is positive.

(iv) Find the range of values of k for which the optimal point is the same as in part **(iii)**. [3]

4 **Answer this question on the insert provided.**

The vertices in the network below represent the rooms in a house. The arcs represent routes between rooms, and the weights on the arcs represent distances in metres.



(i) On the diagram in the insert, use Dijkstra's algorithm to find the shortest path from A to K. You must show your working, including temporary labels, permanent labels and the order in which permanent labels are assigned. Write down the route of the shortest path from A to K and give its length in metres. [7]

A locked door blocks the route CJ, so this arc cannot be used.

(ii) Use your answer to part **(i)** to find the route of the shortest path from A to J and its length in metres. [2]

(iii) Alterations mean that the length of route FJ changes from its current value of 5 metres. By how much would it have to change if the route of the shortest path from A to J, not using CJ, changes from that found in part **(ii)**? [2]

June 2008

5 Laura is booking buses to transport students home from a college party. She wants to book four buses to travel to Easton and five buses to travel to Weston. She contacts the local bus companies to ask about availability and cost. This information is summarised in the table below.

Company	Number of buses available	Cost per bus to Easton	Cost per bus to Weston
Anywhere Autos (*A*)	3	£250	£250
Busy Buses (*B*)	3	£200	£140
County Coaches (*C*)	3	£300	£280

Suppose that Laura books x buses to travel to Easton from company A and y buses to travel to Easton from company B.

(i) **Copy and complete** the following table to show, in terms of x and y, how many buses Laura books from each company to each town and show that the total cost is £$(2090 - 20x + 40y)$. [5]

	E	W
A	x	
B	y	
C		$x + y - 1$

(ii) Laura wants to spend no more than £2150 on the buses.

Show that this leads to the constraint $-x + 2y \leqslant 3$. [1]

When Laura looks at the times that the companies could run the buses, she realises that she will need at least one bus from C to E. This leads to the constraint $x + y \leqslant 3$.

Each bus from A can carry 50 students, each bus from B can carry 40 students and each bus from C can carry 60 students. Laura wants to maximise the number of students who can travel to W.

(iii) Show that this leads to needing to maximise the objective function $x + 2y$. [2]

Laura's problem gives the linear programming problem:

$$\begin{array}{ll} \text{Maximise} & P = x + 2y, \\ \text{subject to} & -x + 2y \leqslant 3, \\ & x + y \leqslant 3, \\ \text{and} & x \geqslant 0,\ y \geqslant 0, \quad \text{with } x \text{ and } y \text{ both integers.} \end{array}$$

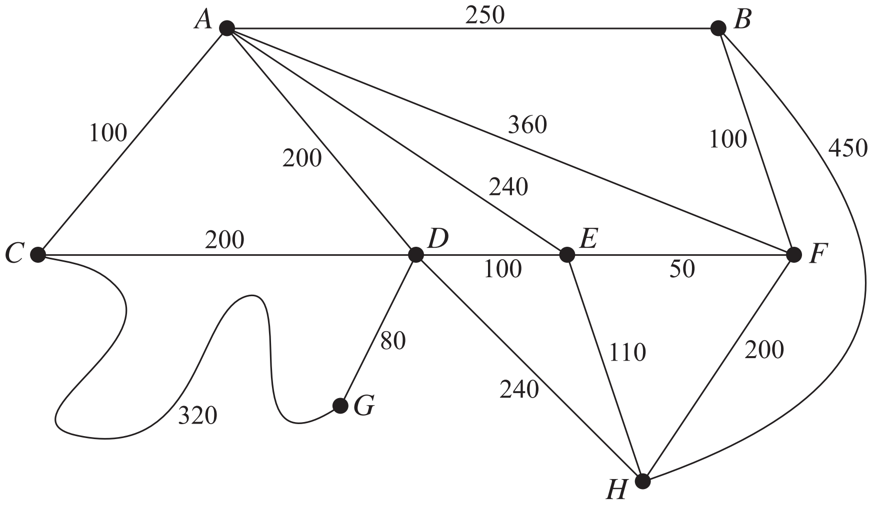
(iv) Represent this problem as an initial Simplex tableau. [2]

(v) Use the Simplex algorithm, pivoting first on a value chosen from the y column, to find the values of x and y at the optimum point. [6]

[Turn over

June 2008

6 The network below represents a simplified map of a forest. The nodes represent locations in the forest and the arcs represent footpaths. The weights on the arcs represent distances, in metres.



(a) Woody the forest ranger wants to start from rangers' hut (H) and walk along every footpath at least once using the shortest possible total distance.

(i) Which standard network problem does Woody need to solve to find the shortest route that covers every arc? [1]

The total length of all the footpaths shown is 3000 metres.

(ii) Use an appropriate algorithm to find the length of the shortest route that Woody can use. Show all your working. (You may find the lengths of shortest paths between nodes by inspection.) [4]

Suppose that, instead, Woody wants to start from the car park (C) and walk along every footpath at least once using the shortest possible total distance.

(iii) What is the length of the shortest route that Woody can use if he starts from the car park? At which node does this route end? [3]

(b) There is a nesting box at each node of the network.

Cyril the squirrel lives in the forest. He wants to start from his drey (D) and check each nesting box, to see whether he has stored any nuts there, before returning to his drey. Cyril is a vain squirrel, so he wants to use the footpaths so that people can see him. However he is also a very lazy squirrel, so he would like to check the boxes in the shortest distance possible.

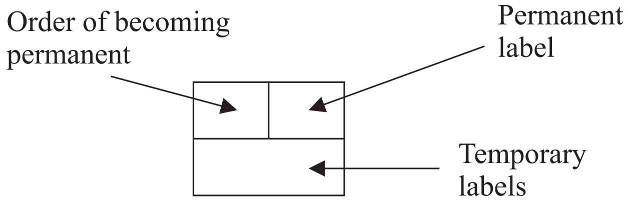
(i) Apply the nearest neighbour method starting at node D to find a tour through all the nodes that starts and ends at D. Calculate the total weight of this tour. Explain why the nearest neighbour method fails if you start at node A. [3]

(ii) Construct a minimum spanning tree by using Prim's algorithm on the reduced network formed by deleting node A and all the arcs that are directly joined to node A. **Start building your tree at node *B*.** (You do *not* need to represent the network as a matrix.) Give the order in which nodes are added to your tree and draw a diagram to show the arcs in your tree. Calculate the total weight of your tree. [5]

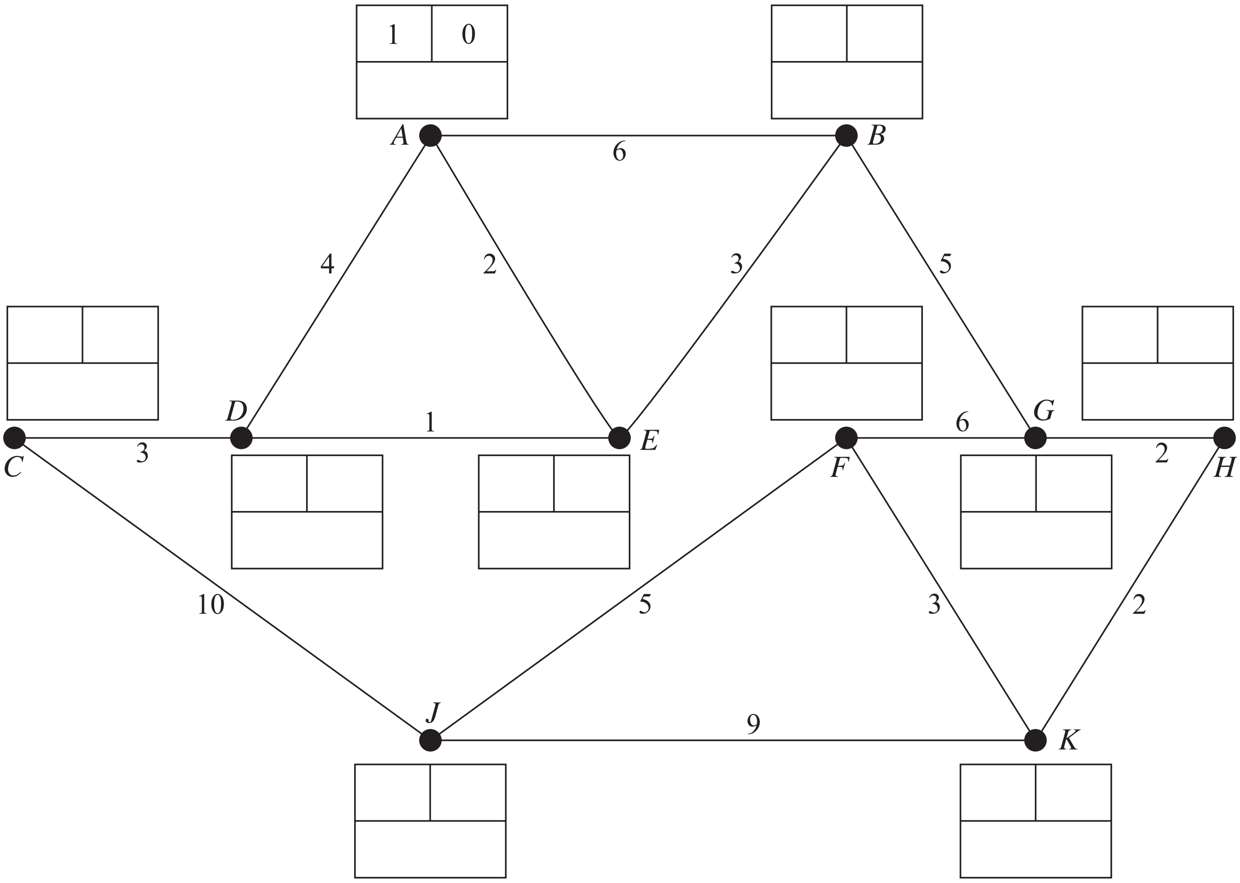
(iii) From your previous answers, what can you say about the shortest possible distance that Cyril must travel to visit each nesting box and return home to his drey? [2]

4 (i)

Key:



Do not cross out your working values (temporary labels)



Route of shortest path from A to K = ...

Length of shortest path from A to K = ... metres

(ii) ....

Route of shortest path from A to J, without using CJ = ...

Length of path = ... metres

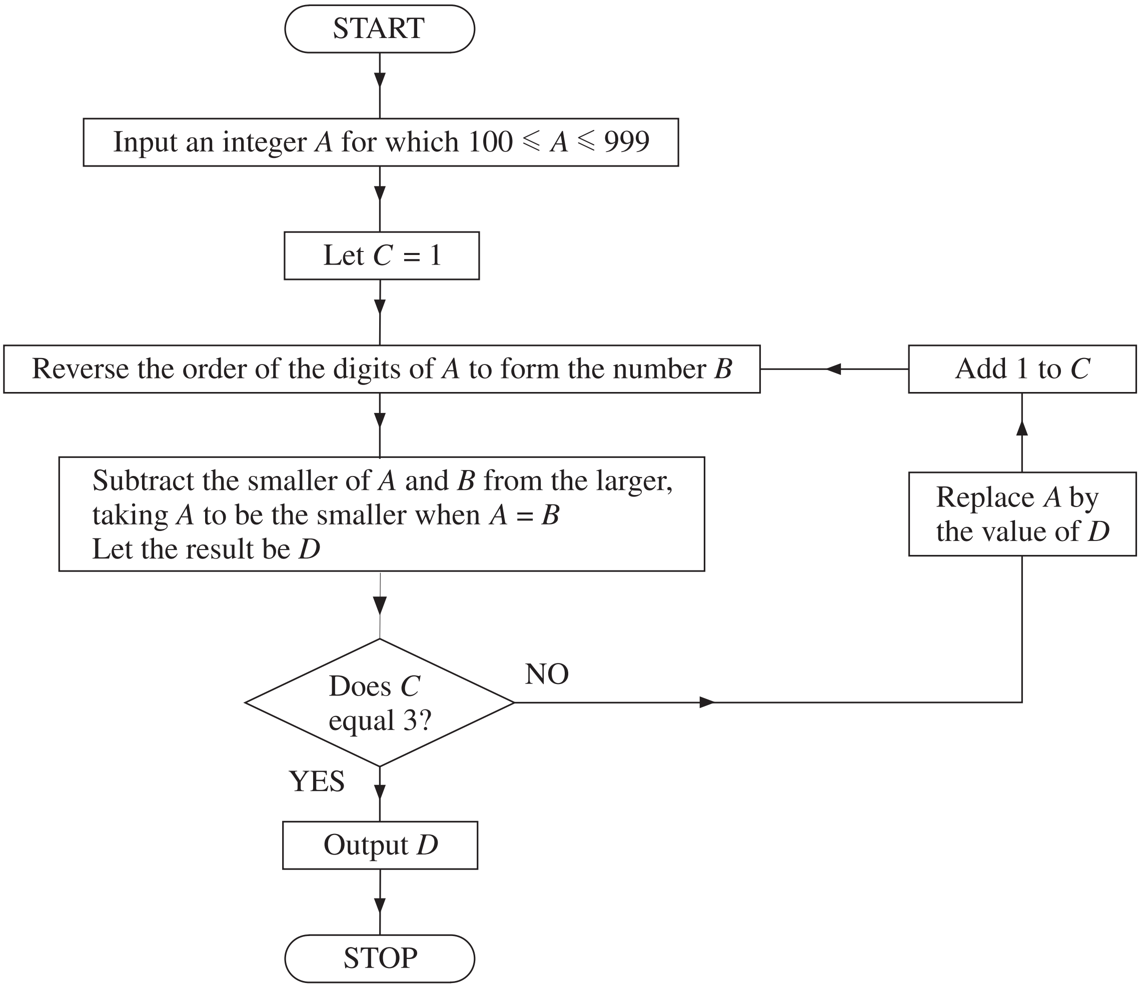
(iii) ....

....

Length of FJ must change by .... metres

Jan 2009

1 The flow chart shows an algorithm for which the input is a three-digit positive integer.



(i) Trace through the algorithm using the input $A = 614$ to show that the output is 297. Write down the values of A, B, C and D in each pass through the algorithm. **[4]**

(ii) What is the output when $A = 616$? **[1]**

(iii) Explain why the counter C is needed. **[1]**

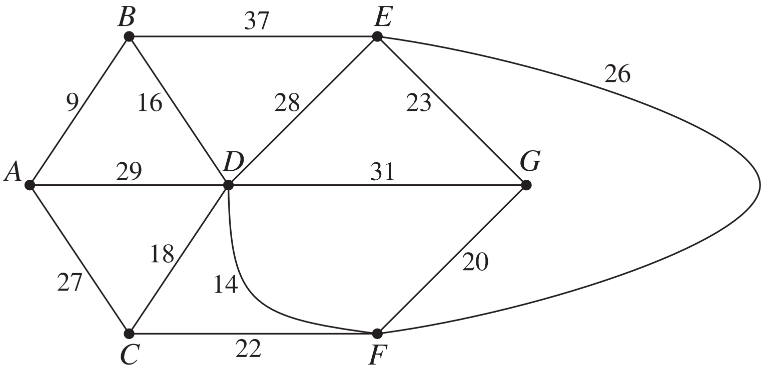
2 **(i)** Draw a graph with five vertices of orders 1, 2, 2, 3 and 4. **[2]**

(ii) State whether the graph from part **(i)** is Eulerian, semi-Eulerian or neither. Explain how you know which it is. **[2]**

(iii) Explain why a graph with five vertices of orders 1, 2, 2, 3 and 4 cannot be a tree. **[2]**

Jan 2009

3 Answer this question on the insert provided.



(i) This diagram shows a network. The insert has a copy of this network together with a list of the arcs, sorted into increasing order of weight. Use Kruskal's algorithm on the insert to find a minimum spanning tree for this network. Draw your tree and give its total weight. **[5]**

(ii) Use your answer to part **(i)** to find the weight of a minimum spanning tree for the network with vertex E, and all the arcs joined to E, removed. Hence find a lower bound for the travelling salesperson problem on the original network. **[3]**

(iii) Show that the nearest neighbour method, starting from vertex A, fails on the original network. **[2]**

(iv) Apply the nearest neighbour method, starting from vertex B, to find an upper bound for the travelling salesperson problem on the original network. **[3]**

(v) Apply Dijkstra's algorithm to the copy of the network in the insert to find the least weight path from A to G. State the weight of the path and give its route. **[6]**

(vi) The sum of the weights of all the arcs is 300.

Apply the route inspection algorithm, showing all your working, to find the weight of the least weight closed route that uses every arc at least once. The weights of least weight paths from vertex A should be found using your answer to part **(v)**; the weights of other such paths should be determined by inspection. **[4]**

Jan 2009

4 Answer this question on the insert provided.

The list of numbers below is to be sorted into **decreasing** order using shuttle sort.

21 76 65 13 88 62 67 28 34

(i) How many passes through shuttle sort will be required to sort the list? **[1]**

After the first pass the list is as follows.

76 21 65 13 88 62 67 28 34

(ii) State the number of comparisons and the number of swaps that were made in the first pass. **[1]**

(iii) Write down the list after the second pass. State the number of comparisons and the number of swaps that were used in making the second pass. **[2]**

(iv) Complete the table in the insert to show the results of the remaining passes, recording the number of comparisons and the number of swaps made in each pass. You may not need all the rows of boxes printed. **[6]**

When the original list is sorted into decreasing order using bubble sort there are 30 comparisons and 17 swaps.

(v) Use your results from part **(iv)** to compare the efficiency of these two methods in this case. **[2]**

Jan 2009

5 Katie makes and sells cookies.

Each batch of plain cookies takes 8 minutes to prepare and then 12 minutes to bake.
Each batch of chocolate chip cookies takes 12 minutes to prepare and then 12 minutes to bake.
Each batch of fruit cookies takes 10 minutes to prepare and then 12 minutes to bake.

Katie can only bake one batch at a time. She has the use of the kitchen, including the oven, for at most 1 hour.

(i) Each batch of cookies must be prepared before it is baked. By considering the maximum time available for baking the cookies, explain why Katie can make at most 4 batches of cookies. **[2]**

Katie models the constraints as

$$\begin{aligned} x + \ \ y + \ \ z &\leqslant 4, \\ 4x + 6y + 5z &\leqslant 24, \\ x \geqslant 0,\ y \geqslant 0,\ z &\geqslant 0, \end{aligned}$$

where x is the number of batches of plain cookies, y is the number of batches of chocolate chip cookies and z is the number of batches of fruit cookies that Katie makes.

(ii) Each batch of cookies that Katie prepares must be baked within the hour available. By considering the maximum time available for preparing the cookies, show how the constraint $4x + 6y + 5z \leqslant 24$ was formed. **[2]**

(iii) In addition to the constraints, what other restriction is there on the values of x, y and z? **[1]**

Katie will make £5 profit on each batch of plain cookies, £4 on each batch of chocolate chip cookies and £3 on each batch of fruit cookies that she sells. Katie wants to maximise her profit.

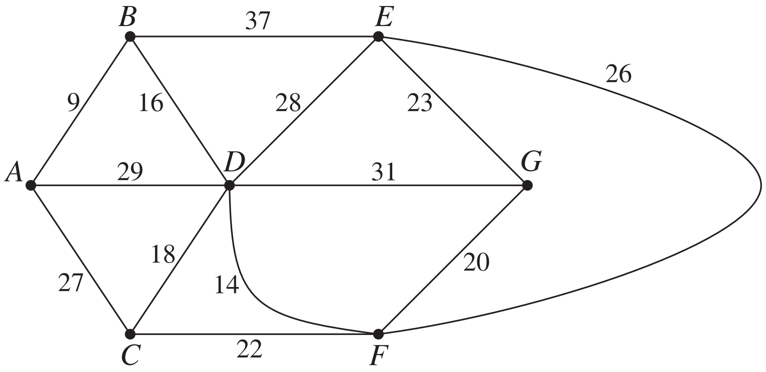
(iv) Write down an expression for the objective function to be maximised. State any assumption that you have made. **[2]**

(v) Represent Katie's problem as an initial Simplex tableau. Perform **one** iteration of the Simplex algorithm, choosing to pivot on an element from the x-column. Show how each row was obtained. Write down the number of batches of cookies of each type and the profit at this stage. **[10]**

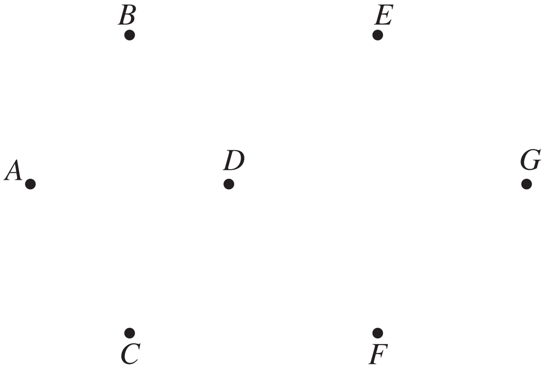
After carrying out market research, Katie decides that she will not make fruit cookies. She also decides that she will make at least twice as many batches of chocolate chip cookies as plain cookies.

(vi) Represent the constraints for Katie's new problem graphically and calculate the coordinates of the vertices of the feasible region. By testing suitable integer-valued coordinates, find how many batches of plain cookies and how many batches of chocolate chip cookies Katie should make to maximise her profit. Show your working. **[8]**

3 **(i)**



$AB = 9$
$DF = 14$
$BD = 16$
$CD = 18$
$FG = 20$
$CF = 22$
$EG = 23$
$EF = 26$
$AC = 27$
$DE = 28$
$AD = 29$
$DG = 31$
$BE = 37$



Total weight of arcs in minimum spanning tree = ...

(ii) Weight of spanning tree for vertices A, B, C, D, F and G only = ...

....

Lower bound for travelling salesperson problem on original network = ...

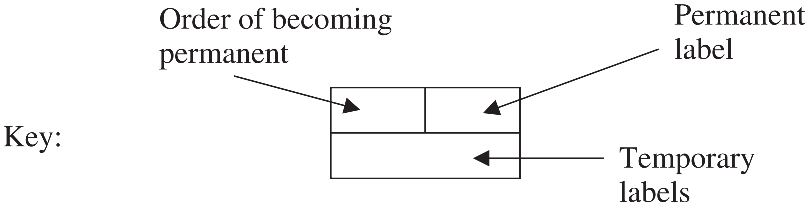
(iii) ....

....

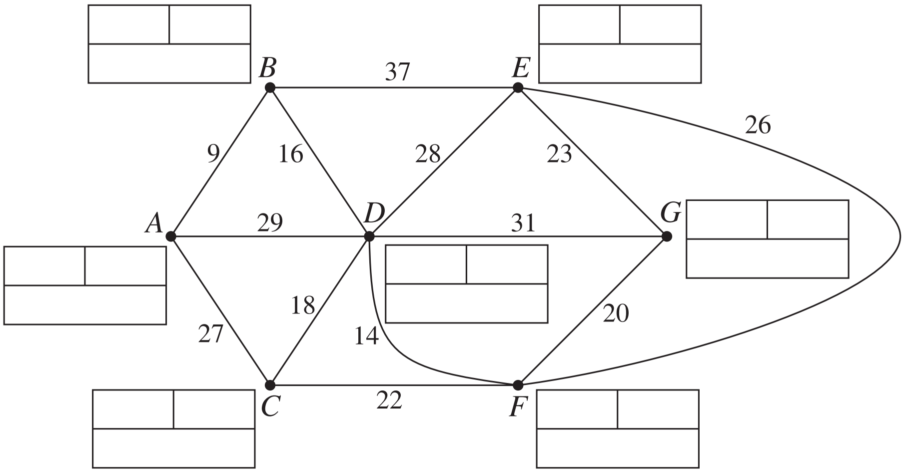
(iv) Nearest neighbour gives B – ....

Upper bound for travelling salesperson problem on original network = ...

(v)



Do not cross out your working values (temporary labels)



Weight = ...

Route = ...

(vi) ....

....

....

....

....

....

4 **(i)** .... passes

(ii) .... comparisons and .... swaps

(iii)

.... comparisons and .... swaps

(iv)

									Comp	Swap

(v) .... is the more efficient method in this case

because ....

....

....

OCR RECOGNISING ACHIEVEMENT

1 The memory requirements, in KB, for eight computer files are given below.

43	172	536	17	314	462	220	231

The files are to be grouped into folders. No folder is to contain more than 1000 KB, so that the folders are small enough to transfer easily between machines.

(i) Use the first-fit method to group the files into folders. **[3]**

(ii) Use the first-fit decreasing method to group the files into folders. **[3]**

First-fit decreasing is a quadratic order algorithm.

(iii) A computer takes 1.3 seconds to apply first-fit decreasing to a list of 500 numbers. Approximately how long will it take to apply first-fit decreasing to a list of 5000 numbers? **[2]**

2 **(i)** Explain why it is impossible to draw a graph with four vertices in which the vertex orders are 1, 2, 3 and 3. **[1]**

A *simple* graph is one in which any two vertices are directly joined by at most one arc and no vertex is directly joined to itself.

A *connected* graph is one in which every vertex is joined, directly or indirectly, to every other vertex.

A *simply connected* graph is one that is both simple and connected.

(ii) **(a)** Draw a graph with five vertices of orders 1, 1, 2, 2 and 4 that is neither simple nor connected. **[2]**

(b) Explain why your graph from part **(a)** is not semi-Eulerian. **[1]**

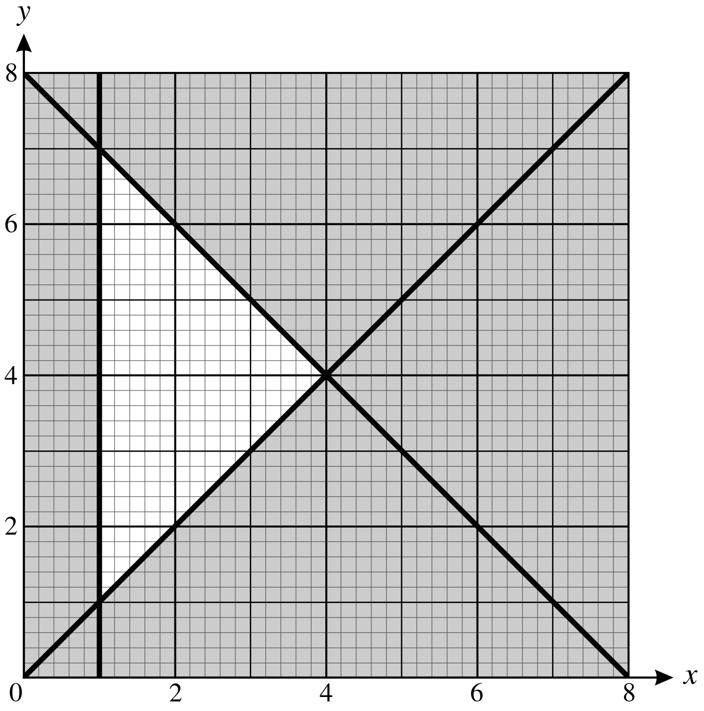
(c) Draw a semi-Eulerian graph with five vertices of orders 1, 1, 2, 2 and 4. **[1]**

Six people (Ann, Bob, Caz, Del, Eric and Fran) are represented by the vertices of a graph. Each pair of vertices is joined by an arc, forming a complete graph. If an arc joins two vertices representing people who have met it is coloured blue, but if it joins two vertices representing people who have not met it is coloured red.

(iii) **(a)** Explain why the vertex corresponding to Ann must be joined to at least three of the others by arcs that are the same colour. **[2]**

(b) Now assume that Ann has met Bob, Caz and Del. Bob, Caz and Del may or may not have met one another. Explain why the graph must contain at least one triangle of arcs that are all the same colour. **[2]**

3 The constraints of a linear programming problem are represented by the graph below. The feasible region is the unshaded region, including its boundaries.



(i) Write down the inequalities that define the feasible region. **[4]**

(ii) Write down the coordinates of the three vertices of the feasible region. **[2]**

The objective is to maximise $2x + 3y$.

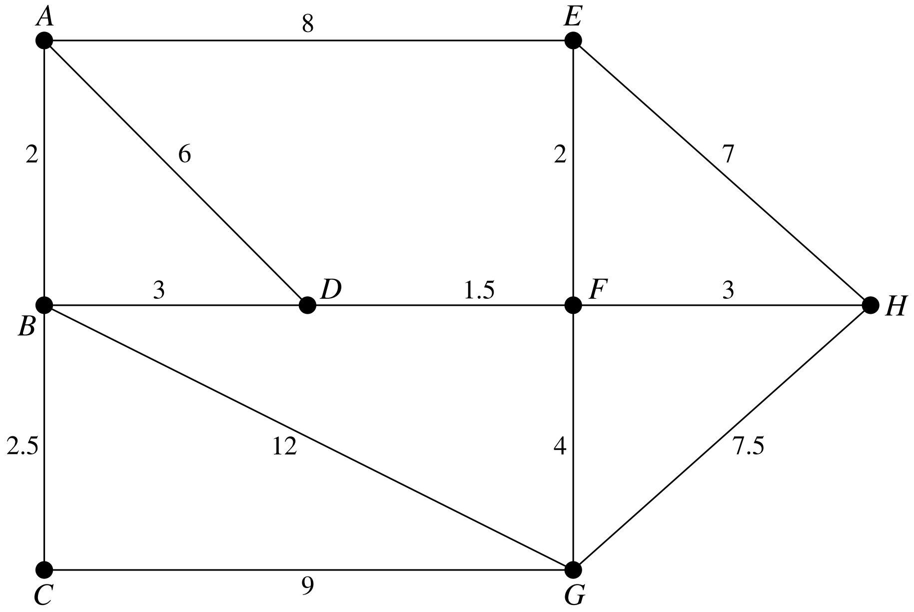
(iii) Find the values of x and y at the optimal point, and the corresponding maximum value of $2x + 3y$. **[3]**

The objective is changed to maximise $2x + ky$, where k is positive.

(iv) Find the range of values of k for which the optimal point is the same as in part **(iii)**. **[2]**

4 Answer this question on the insert provided.

The vertices in the network below represent the junctions between main roads near Ayton (A). The arcs represent the roads and the weights on the arcs represent distances in miles.



(i) On the diagram in the insert, use Dijkstra's algorithm to find the shortest path from A to H. You must show your working, including temporary labels, permanent labels and the order in which permanent labels are assigned. Write down the route of the shortest path from A to H and give its length in miles. **[7]**

Simon is a highways surveyor. He needs to check that there are no potholes in any of the roads. He will start and end at Ayton.

(ii) Which standard network problem does Simon need to solve to find the shortest route that uses every arc? **[1]**

The total weight of all the arcs is 67.5 miles.

(iii) Use an appropriate algorithm to find the length of the shortest route that Simon can use. Show all your working. (You may find the lengths of shortest paths between nodes by using your answer to part **(i)** or by inspection.) **[5]**

Suppose that, instead, Simon wants to find the shortest route that uses every arc, starting from A and ending at H.

(iv) Which arcs does Simon need to travel twice? What is the length of the shortest route that he can use? **[2]**

[This question continues on the next page.]

June 2009

There is a set of traffic lights at each junction. Simon's colleague Amber needs to check that all the traffic lights are working correctly. She will start and end at the same junction.

(v) Show that the nearest neighbour method fails on this network if it is started from A. **[1]**

(vi) Apply the nearest neighbour method starting from C to find an upper bound for the distance that Amber must travel. **[3]**

(vii) Construct a minimum spanning tree by using Prim's algorithm on the reduced network formed by deleting node A and all the arcs that are directly joined to node A. **Start building your tree at node *B*.** (You do *not* need to represent the network as a matrix.) Mark the arcs in your tree on the diagram in the insert.

Give the order in which nodes are added to your tree and calculate the total weight of your tree. Hence find a lower bound for the distance that Amber must travel. **[6]**

5 *Badgers* is a small company that makes badges to customers' designs. Each badge must pass through four stages in its production: printing, stamping out, fixing pin and checking. The badges can be laminated, metallic or plastic.

The times taken **for 100 badges** of each type to pass through each of the stages and the profits that *Badgers* makes **on every 100 badges** are shown in the table below. The table also shows the total time available for each of the production stages.

	Printing (seconds)	Stamping out (seconds)	Fixing pin (seconds)	Checking (seconds)	Profit (£)
Laminated	15	5	50	100	4
Metallic	15	8	50	50	3
Plastic	30	10	50	20	1
Total time available	9000	3600	25 000	10 000	

Suppose that the company makes x hundred laminated badges, y hundred metallic badges and z hundred plastic badges.

(i) Show that the printing time leads to the constraint $x + y + 2z \leqslant 600$. Write down and simplify constraints for the time spent on each of the other production stages. **[4]**

(ii) What other constraint is there on the values of x, y and z? **[1]**

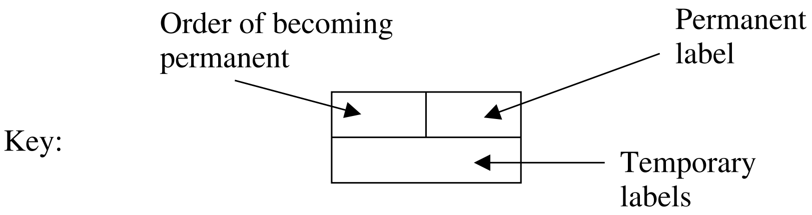
The company wants to maximise the profit from the sale of badges.

(iii) Write down an appropriate objective function, to be maximised. **[1]**

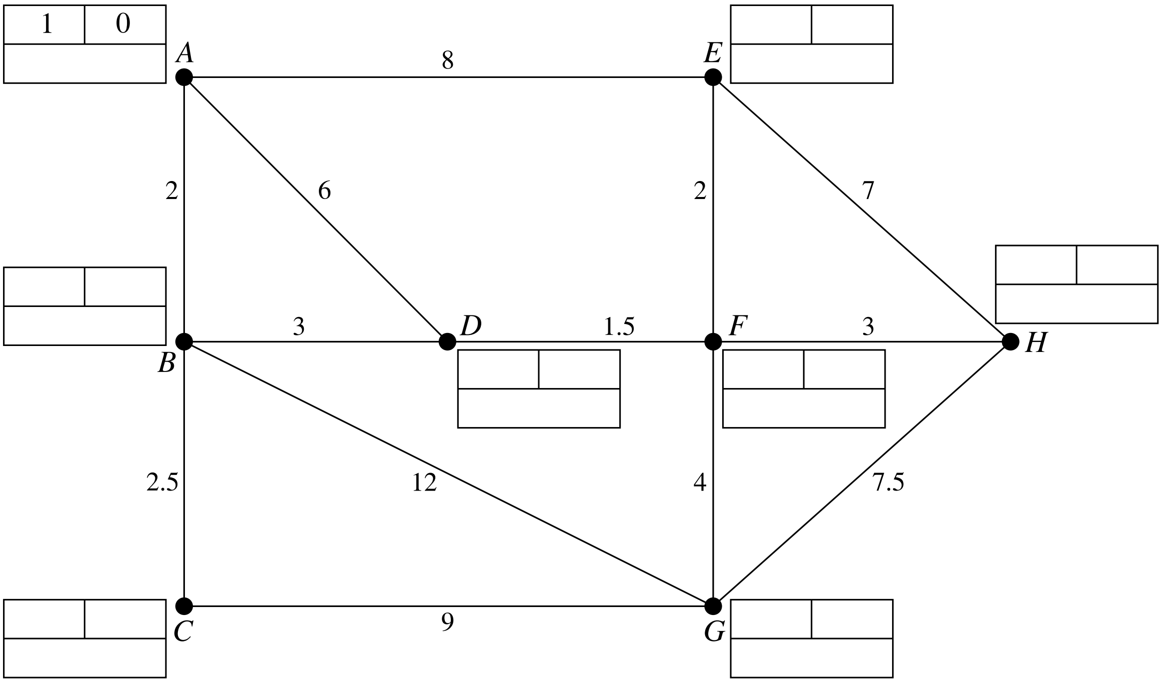
(iv) Represent *Badgers*' problem as an initial Simplex tableau. **[4]**

(v) Use the Simplex algorithm, pivoting first on a value chosen from the x-column and then on a value chosen from the y-column. Interpret your solution and the values of the slack variables in the context of the original problem. **[9]**

4 **(i)**



Do not cross out your working values (temporary labels)



Route of shortest path from A to H = ...

Length of shortest path from A to H = ... miles

(ii) ....

(iii) ....

....

....

....

....

Length of shortest route = ... miles

(iv) Repeat arcs ....

Length of shortest route = ... miles

(v) ....

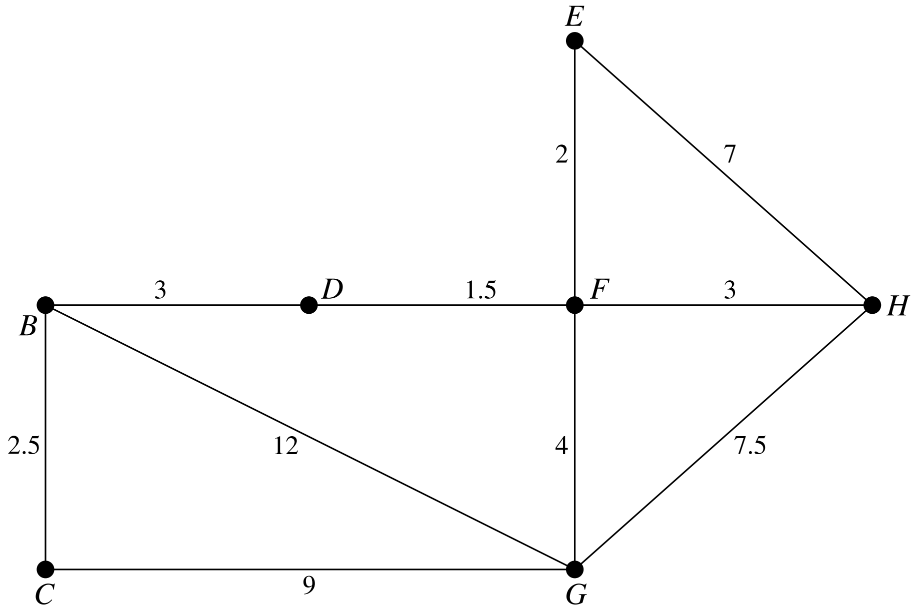
....

(vi) ....

....

Upper bound = ... miles

(vii)



Order of adding nodes to tree: ....

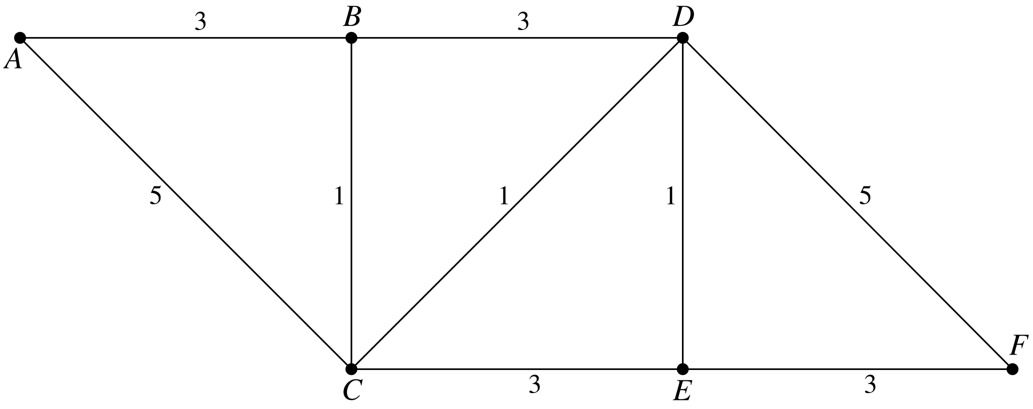
Total weight = ... miles

....

....

Lower bound = ... miles

1 Answer this question on the insert provided.



(i) Apply Dijkstra's algorithm to the copy of this network in the insert to find the least weight path from A to F. State the route of the path and give its weight. **[5]**

(ii) Apply the route inspection algorithm, showing all your working, to find the weight of the least weight closed route that uses every arc at least once. Write down a closed route that has this least weight. **[4]**

An extra arc is added, joining B to E, with weight 2.

(iii) Write down the new least weight path from A to F. Explain why the new least weight closed route, that uses every arc at least once, has no repeated arcs. **[2]**

2 A *simple* graph is one in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself.

A *connected* graph is one in which every vertex is joined, directly or indirectly, to every other vertex.

A *simply connected* graph is one that is both simple and connected.

(i) Explain why there is no simply connected graph with exactly five vertices each of which is connected to exactly three others. **[1]**

(ii) A simply connected graph has five vertices A, B, C, D and E, in which A has order 4, B has order 2, C has order 3, D has order 3 and E has order 2. Explain how you know that the graph is semi-Eulerian and write down a semi-Eulerian trail on this graph. **[2]**

A network is formed from the graph in part **(ii)** by weighting the arcs as given in the table below.

	A	B	C	D	E
A	–	5	3	8	2
B	5	–	6	–	–
C	3	6	–	7	–
D	8	–	7	–	9
E	2	–	–	9	–

(iii) Apply Prim's algorithm to the network, showing all your working, starting at vertex A. Draw the resulting tree and state its total weight. **[3]**

A sixth vertex, F, is added to the network using arcs CF and DF, each of which has weight 6.

(iv) Use your answer to part **(iii)** to write down a lower bound for the length of the minimum tour that visits every vertex of the extended network, finishing where it starts. Apply the nearest neighbour method, starting from vertex A, to find an upper bound for the length of this tour. Explain why the nearest neighbour method fails if it is started from vertex F. **[4]**

3 Maggie is a personal trainer. She has twelve clients who want to lose weight. She decides to put some of her clients on weight loss programme X, some on programme Y and the rest on programme Z. Each programme involves a strict diet; in addition programmes X and Y involve regular exercise at Maggie's home gym. The programmes each last for one month.

In addition to the diet, clients on programme X spend 30 minutes each day on the spin cycle, 10 minutes each day on the rower and 20 minutes each day on free weights. At the end of one month they can each expect to have lost 9 kg more than a client on just the diet.

In addition to the diet, clients on programme Y spend 10 minutes each day on the spin cycle and 30 minutes each day on free weights; they do not use the rower. At the end of one month they can each expect to have lost 6 kg more than a client on just the diet.

Because of other clients who use Maggie's home gym, the spin cycle is available for the weight loss clients for 180 minutes each day, the rower for 40 minutes each day and the free weights for 300 minutes each day. Only one client can use each piece of apparatus at any one time.

Maggie wants to decide how many clients to put on each programme to maximise the total expected weight loss at the end of the month. She models the objective as follows.

$$\text{Maximise } P = 9x + 6y$$

(i) What do the variables x and y represent? **[1]**

(ii) Write down and simplify the constraints on the values of x and y from the availability of each of the pieces of apparatus. **[3]**

(iii) What other constraints and restrictions apply to the values of x and y? **[1]**

(iv) Use a graphical method to represent the feasible region for Maggie's problem. You should use graph paper and choose scales so that the feasible region can be clearly seen. Hence determine how many clients should be put on each programme. **[6]**

4 Jack and Jill are packing food parcels. The boxes for the food parcels can each carry up to 5000 g in weight and can each hold up to 30 000 cm^3 in volume.

The number of each item to be packed, their dimensions and weights are given in the table below.

Item type	*A*	*B*	*C*	*D*
Number to be packed	15	8	3	4
Length (cm)	10	40	20	10
Width (cm)	10	30	50	40
Height (cm)	10	20	10	10
Volume (cm^3)	1000	24 000	10 000	4000
Weight (g)	1000	250	300	400

Jill tries to pack the items by weight using the first-fit decreasing method.

(i) List the 30 items in order of decreasing weight and hence show Jill's packing. Explain why Jill's packing is not possible. **[5]**

Jack tries to pack the items by volume using the first-fit decreasing method.

(ii) List the 30 items in order of decreasing volume and hence show Jack's packing. Explain why Jack's packing is not possible. **[5]**

(iii) Give another reason why a packing may not be possible. **[1]**

5 Consider the following LP problem.

$$\begin{aligned} &\text{Minimise} && 2a - 3b + c + 18, \\ &\text{subject to} && a + b - c \geqslant 14, \\ &&& -2a + 3c \leqslant 50, \\ &&& 10 + 4a \geqslant 5b, \\ &\text{and} && a \leqslant 20,\ b \leqslant 10,\ c \leqslant 8. \end{aligned}$$

(i) By replacing a by $20 - x$, b by $10 - y$ and c by $8 - z$, show that the problem can be expressed as follows.

$$\begin{aligned} &\text{Maximise} && 2x - 3y + z, \\ &\text{subject to} && x + y - z \leqslant 8, \\ &&& 2x - 3z \leqslant 66, \\ &&& 4x - 5y \leqslant 40, \\ &\text{and} && x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0. \end{aligned}$$ **[3]**

(ii) Represent the problem as an initial Simplex tableau. Perform **one** iteration of the Simplex algorithm. Explain how the choice of pivot was made and show how each row was obtained. Write down the values of x, y and z at this stage. Hence write down the corresponding values of a, b and c. **[11]**

(iii) If, additionally, the variables a, b and c are non-negative, what additional constraints are there on the values of x, y and z? **[2]**

6 Answer this question on the insert provided.

In this question you will need the result: $1 + 2 + \ldots + k = \frac{1}{2}k(k + 1)$.

Dominic is writing a computer program to carry out Kruskal's algorithm. He starts by writing a procedure that enables him to input arcs and their weights, for example $\boxed{A \mid 8 \mid B}$ would represent an arc joining A to B of weight 8.

(i) If Dominic uses a network with five vertices, what is the greatest number of arcs that he needs to input? What is the greatest number of arcs for a network with n vertices? **[2]**

He then uses shuttle sort to sort the inputs into order of increasing weight.

(ii) (a) For a network with five vertices, write down

- the maximum number of passes
- the maximum number of comparisons in the first, second and third passes
- the maximum total number of comparisons. **[3]**

(b) Show that the maximum total number of comparisons for a network with n vertices is $\frac{1}{4}n(n-1)\left(\frac{1}{2}n(n-1)-1\right)$. **[2]**

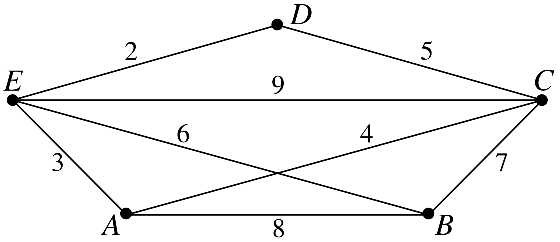
Dominic then sets up four memory areas. *M1* is for the vertices that are in the tree, *M2* is for the arcs that are in the tree and *M3* is for the vertices that are not in the tree. Initially, *M1*and *M2* are empty and *M3* contains a list of all the vertices. Dominic stores the sorted list of arcs and their weights in *M4*.

The first arc on the sorted list is added to the tree, the vertices at its ends are transferred from *M3* to *M1* and the arc is transferred from *M4* to *M2*.

The arc that is now first in *M4* is considered. Each of the two vertices that define the arc is compared with every entry in *M3*. If either of the vertices appears in *M3*, the arc is added to the tree by transferring the vertices at its ends from *M3* to *M1* and transferring the arc from *M4* to *M2*. If neither of the vertices appears in *M3*, the arc is just deleted from *M4*.

This is continued until *M4* is empty.

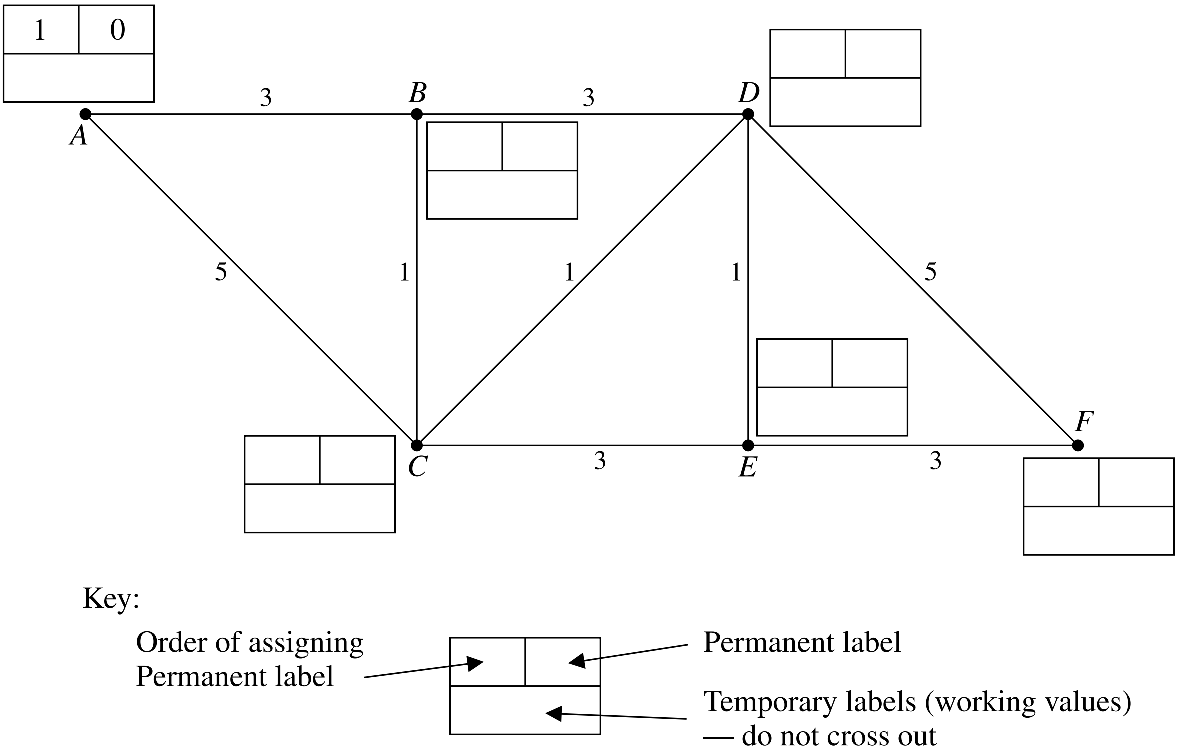
(iii) The insert shows the start of Dominic's program for the network shown below.



Complete the working on the table in the insert. **[4]**

(iv) Dominic's program has quartic order (order n^4). Dominic's program takes 30 seconds to process an input from a network with 100 vertices. Approximately how long would it take to process an input from a network with 500 vertices? **[2]**

1 **(i)**



....

....

Path ....

Weight ....

(ii) ....

....

Weight ....

Route ....

(iii) ....

....

....

6 **(i)** Greatest number of arcs

for a network with five vertices = ...

for a network with *n* vertices = ...

(ii) **(a)** For a network with five vertices

maximum number of passes = ...

maximum number of comparisons

in the first pass = ...

in the second pass = ...

in the third pass = ...

maximum total number of comparisons = ...

(b) For a network with *n* vertices

maximum total number of comparisons = ...

....

....

(iii)

M1 Vertices in tree	***M2*** Arcs in tree	***M3*** Vertices not in tree
		A B C D E
D E	*D* 2 *E*	*A B C*
	D 2 *E*	
	D 2 *E*	
	D 2 *E*	

M4 Sorted list
~~*D* 2 *E*~~
A 3 *E*
A 4 *C*
C 5 *D*
B 6 *E*
B 7 *C*
A 8 *B*
C 9 *E*

(iv) ....

....

1 Owen and Hari each want to sort the following list of marks into decreasing order.

31 28 75 87 42 43 70 56 61 95

(i) Owen uses bubble sort, starting from the left-hand end of the list.

(a) Show the result of the first pass through the list. Record the number of comparisons and the number of swaps used in this first pass. Which marks, if any, are guaranteed to be in their correct final positions after the first pass? **[4]**

(b) Write down the list at the end of the second pass of bubble sort. **[1]**

(c) How many more passes are needed to get the value 95 to the start of the list? **[1]**

(ii) Hari uses shuttle sort, starting from the left-hand end of the list.

Show the results of the first and the second pass through the list. Record the number of comparisons and the number of swaps used in each of these passes. **[4]**

(iii) Explain why, for this particular list, the total number of comparisons will be greater using bubble sort than using shuttle sort. **[2]**

Shuttle sort is a quadratic order algorithm.

(iv) If it takes Hari 20 seconds to sort a list of ten marks using shuttle sort, approximately how long will it take Hari to sort a list of fifty marks? **[2]**

2 A *simple* graph is one in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself.

A *connected* graph is one in which every vertex is joined, directly or indirectly, to every other vertex.

A *simply connected* graph is one that is both simple and connected.

(i) Explain why it is impossible to draw a graph with exactly three vertices in which the vertex orders are 2, 3 and 4. **[1]**

(ii) Draw a graph with exactly four vertices of orders 1, 2, 3 and 4 that is neither simple nor connected. **[2]**

(iii) Explain why there is no simply connected graph with exactly four vertices of orders 1, 2, 3 and 4. State which of the properties 'simple' and 'connected' cannot be achieved. **[2]**

(iv) A simply connected Eulerian graph has exactly five vertices.

(a) Explain why there cannot be exactly three vertices of order 4. **[1]**

(b) By considering the vertex orders, explain why there are only four such graphs. Draw an example of each. **[3]**

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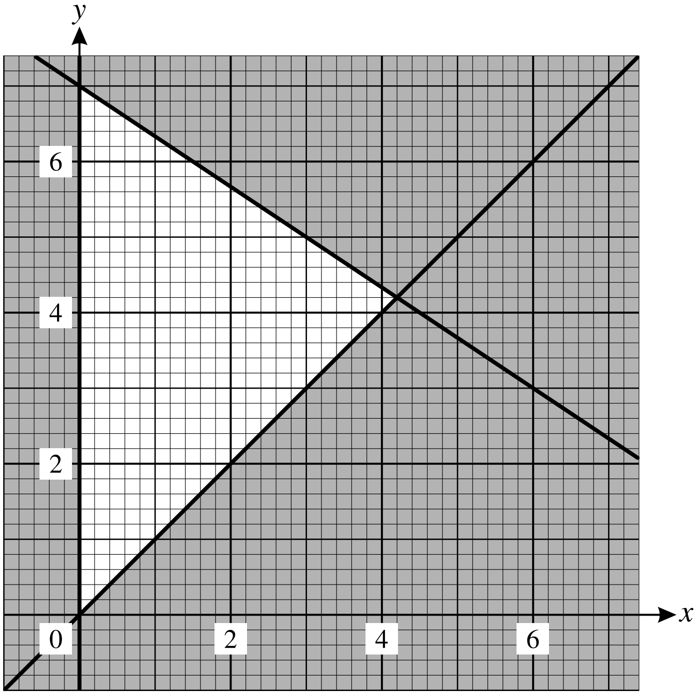
3 The constraints of a linear programming problem are represented by the graph below. The feasible region is the unshaded region, including its boundaries.



(i) Write down the inequalities that define the feasible region. **[3]**

The objective is to maximise $P_1 = x + 6y$.

(ii) Find the values of x and y at the optimal point, and the corresponding value of P_1. **[3]**

The objective is changed to maximise $P_k = kx + 6y$, where k is positive.

(iii) Calculate the coordinates of the optimal point, and the corresponding value of P_k when the optimal point is not the same as in part **(ii)**. **[2]**

(iv) Find the range of values of k for which the point identified in part **(ii)** is still optimal. **[2]**

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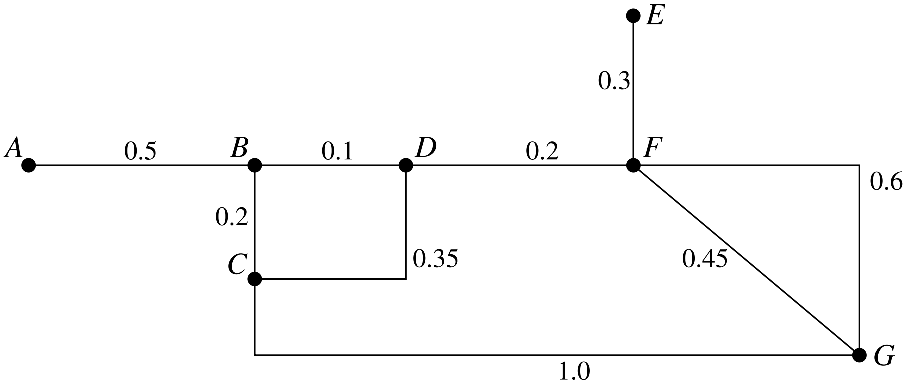
4 The network below represents a small village. The arcs represent the streets and the weights on the arcs represent distances in km.



(i) Use Dijkstra's algorithm to find the shortest path from A to G. You must show your working, including temporary labels, permanent labels and the order in which permanent labels are assigned. Write down the route of the shortest path from A to G. **[5]**

Hannah wants to deliver newsletters along every street; she will start and end at A.

(ii) Which standard network problem does Hannah need to solve to find the shortest route that uses every arc? **[1]**

The total weight of all the arcs is 3.7 km.

(iii) Hannah knows that she will need to travel AB and EF twice, once in each direction. With this information, use an appropriate algorithm to find the length of the shortest route that Hannah can use. Show all your working. (You may find the lengths of shortest paths between vertices by inspection.) **[5]**

There are street name signs at each vertex except for A and E. Hannah's friend Peter wants to check that the signs have not been vandalised. He will start and end at B.

The table below shows the complete set of shortest distances between vertices B, C, D, F and G.

	B	C	D	F	G
B	–	0.2	0.1	0.3	0.75
C	0.2	–	0.3	0.5	0.95
D	0.1	0.3	–	0.2	0.65
F	0.3	0.5	0.2	–	0.45
G	0.75	0.95	0.65	0.45	–

(iv) Apply the nearest neighbour method to this table, starting from B, to find an upper bound for the distance that Peter must travel. **[2]**

(v) Apply Prim's algorithm to the matrix formed by deleting the row and column for vertex G from the table. Start building your tree at vertex B.

Draw your tree. Give the order in which vertices are built into your tree and calculate the total weight of your tree. Hence find a lower bound for the distance that Peter must travel. **[4]**

5 Jenny is making three speciality smoothies for a party: *fruit salad*, *ginger zinger* and *high C*.

Each litre of *fruit salad* contains 600 calories and has 120 mg of sugar and 100 mg of vitamin C.
Each litre of *ginger zinger* contains 800 calories and has 80 mg of sugar and 40 mg of vitamin C.
Each litre of *high C* contains 500 calories and has 120 mg of sugar and 120 mg of vitamin C.

Jenny has enough milk to make 5 litres of *fruit salad* or 3 litres of *ginger zinger* or 4 litres of *high C*. This leads to the constraint

$$12x + 20y + 15z \leqslant 60$$

in which x represents the number of litres of *fruit salad*, y represents the number of litres of *ginger zinger* and z represents the number of litres of *high C*.

Jenny wants there to be no more than 5000 calories and no more than 800 mg of sugar in total in the smoothies that she makes.

(i) Use this information to write down and simplify two more constraints on the values of x, y and z, other than that they are non-negative. **[4]**

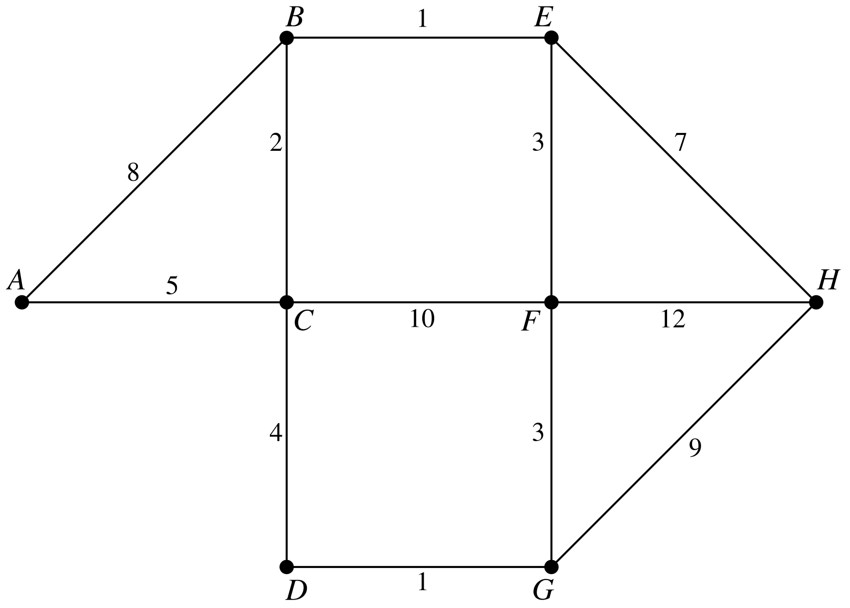
Jenny wants to maximise the total amount of vitamin C in the smoothies. This gives the following objective.

$$\text{Maximise } P = 100x + 40y + 120z$$

(ii) Represent Jenny's problem as an initial Simplex tableau. Use the Simplex algorithm, choosing the first pivot from the z column and showing all your working, to find the optimum. How much of each type of smoothie should Jenny make? **[13]**

(iii) Show that if the first pivot had been chosen from the x column then the optimum would have been achieved in one iteration instead of two. **[5]**

1 In the network below, the arcs represent the roads in Ayton, the vertices represent roundabouts, and the arc weights show the number of traffic lights on each road. Sam is an evening class student at Ayton Academy (A). She wants to drive from the academy to her home (H). Sam hates waiting at traffic lights so she wants to find the route for which the number of traffic lights is a minimum.



(i) Apply Dijkstra's algorithm to find the route that Sam should use to travel from A to H. At each vertex, show the temporary labels, the permanent label and the order of permanent labelling. **[5]**

In the daytime, Sam works for the highways department. After an electrical storm, the highways department wants to check that all the traffic lights are working. Sam is sent from the depot (D) to drive along every road and return to the depot. Sam needs to pass every traffic light, but wants to repeat as few as possible.

(ii) Find the minimum number of traffic lights that must be repeated. Show your working. **[4]**

Suppose, instead, that Sam wants to start at the depot, drive along every road and end at her home, passing every traffic light but repeating as few as possible.

(iii) Find a route on which the minimum number of traffic lights must be repeated. Explain your reasoning. **[3]**

2 Five rooms, A, B, C, D, E, in a building need to be connected to a computer network using expensive cabling. Rob wants to find the cheapest way to connect the rooms by finding a minimum spanning tree for the cable lengths. The length of cable, in metres, needed to connect each pair of rooms is given in the table below.

		Room				
		A	B	C	D	E
	A	–	12	30	15	22
	B	12	–	24	16	30
Room	C	30	24	–	20	25
	D	15	16	20	–	10
	E	22	30	25	10	–

(i) Apply Prim's algorithm in matrix (table) form, starting at vertex A and showing all your working. Write down the order in which arcs were added to the tree. Draw the resulting tree and state the length of cable needed. **[4]**

A sixth room, F, is added to the computer network. The distances from F to each of the other rooms are $AF = 32$, $BF = 29$, $CF = 31$, $DF = 35$, $EF = 30$.

(ii) Use your answer to part **(i)** to write down a lower bound for the length of the minimum tour that visits every vertex of the extended network, finishing where it starts. Apply the nearest neighbour method, starting from vertex A, to find an upper bound for the length of this tour. **[4]**

3 **(i)** Explain why it is impossible to draw a graph with exactly four vertices of orders 1, 2, 3 and 3. **[1]**

A *simple* graph is one in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself.

A *connected* graph is one in which every vertex is joined, directly or indirectly, to every other vertex.

A *simply connected* graph is one that is both simple and connected.

(ii) Explain why there is no simply connected graph with exactly four vertices of orders 1, 1, 2 and 4. **[1]**

(iii) A connected graph has four vertices A, B, C and D, in which A, B and C have order 2 and D has order 4. Explain how you know that the graph is Eulerian. Draw an example of such a graph and write down an Eulerian trail for your graph. **[3]**

A graph has three vertices, A, B and C of orders a, b and c, respectively.

(iv) What restrictions on the values of a, b and c follow from the graph being

(a) simple,

(b) connected,

(c) semi-Eulerian? **[3]**

4 **(i)** Describe carefully how to carry out the first pass through bubble sort when we are using it to sort a list of n numbers into increasing order. State which value is guaranteed to be in its correct final position after the first pass and hence explain how to carry out the second pass on a reduced list. Write down the stopping condition for bubble sort. **[5]**

(ii) Show the list of six values that results at the end of each pass when we use bubble sort to sort this list into increasing order.

3 10 8 2 6 11

You do not need to count the number of comparisons and the number of swaps that are used. **[3]**

Zack wants to cut lengths of wood from planks that are 20 feet long. The following lengths, in feet, are required.

3 10 8 2 6 11

(iii) Use the first-fit method to find a way to cut the pieces. **[2]**

(iv) Use the first-fit decreasing method to find a way to cut the pieces. Give a reason why this might be a more useful cutting plan than that from part **(iii)**. **[2]**

(v) Find a more efficient way to cut the pieces. How many planks will Zack need with this cutting plan and how many cuts will he need to make? **[2]**

5 An online shopping company selects some of its parcels to be checked before posting them. Each selected parcel must pass through three checks, which may be carried out in any order. One person must check the contents, another must check the postage and a third person must check the address.

The parcels are classified according to the type of customer as 'new', 'occasional' or 'regular'. The table shows the time taken, in minutes, for each check on each type of parcel.

	Check contents	Check postage	Check address
New	3	4	3
Occasional	5	3	4
Regular	2	3	3

The manager in charge of checking at the company has allocated each type of parcel a 'value' to represent how useful it is for generating additional income. In suitable units, these values are as follows.

new = 8 points occasional = 7 points regular = 4 points

The manager wants to find out how many parcels of each type her department should check each hour, on average, to maximise the total value. She models this objective as

$$\text{Maximise } P = 8x + 7y + 4z.$$

(i) What do the variables x, y and z represent? **[1]**

(ii) Write down the constraints on the values of x, y and z. **[4]**

The manager changes the value of parcels for regular customers to 0 points.

(iii) Explain what effect this has on the objective and simplify the constraints. **[2]**

(iv) Use a graphical method to represent the feasible region for the manager's new problem. You should choose scales so that the feasible region can be clearly seen. Hence determine the optimal strategy. **[6]**

Now suppose that there is **exactly one hour** available for checking and the manager wants to find out how many parcels of each type her department should check in that hour to maximise the total value. The value of parcels for regular customers is still 0 points.

(v) Find the optimal strategy in this situation. **[3]**

(vi) Give a reason why, even if all the timings and values are correct, the total value may be less than this maximum. **[1]**

Question 6 is printed overleaf.

6 Consider the following LP problem.

$$\begin{array}{ll} \text{Minimise} & 2a - 4b + 5c - 30, \\ \text{subject to} & 3a + 2b - c \geqslant 10, \\ & -2a + 4c \leqslant 35, \\ & 4a - b \leqslant 20, \\ \text{and} & a \leqslant 6,\ b \leqslant 8,\ c \leqslant 10. \end{array}$$

(i) Since $a \leqslant 6$ it follows that $6 - a \geqslant 0$, and similarly for b and c. Let $6 - a = x$ (so that a is replaced by $6 - x$), $8 - b = y$ and $10 - c = z$ to show that the problem can be expressed as

$$\begin{array}{ll} \text{Maximise} & 2x - 4y + 5z, \\ \text{subject to} & 3x + 2y - z \leqslant 14, \\ & 2x - 4z \leqslant 7, \\ & -4x + y \leqslant 4, \\ \text{and} & x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0. \end{array}$$

[3]

(ii) Represent the problem as an initial Simplex tableau. Perform **two** iterations of the Simplex algorithm, showing how each row was obtained. Hence write down the values of a, b and c after two iterations. Find the value of the objective for the original problem at this stage. **[10]**

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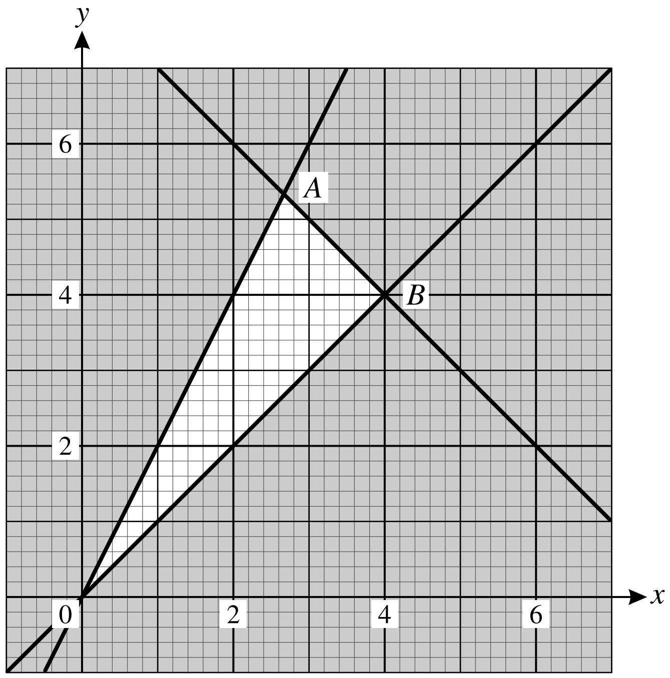
1 The constraints of a linear programming problem are represented by the graph below. The feasible region is the unshaded region, including its boundaries.



(i) Write down the inequalities that define the feasible region. **[2]**

The objective is to maximise $P_m = x + my$, where m is a positive, real-valued constant.

(ii) In the case when $m = 2$, calculate the values of x and y at the optimal point, and the corresponding value of P_2. **[2]**

(iii) **(a)** Write down the values of m for which point A is optimal.

(b) Write down the values of m for which point B is optimal. **[2]**

2 Consider the following algorithm.

```
STEP 1    Input a number $N$
STEP 2    Calculate $R = N \div 2$
STEP 3    Calculate $S = \big((N \div R) + R\big) \div 2$
STEP 4    If $R$ and $S$ are the same when rounded to 2 decimal places, go to STEP 7
STEP 5    Replace $R$ with the value of $S$
STEP 6    Go to STEP 3
STEP 7    Output the value of $R$ correct to 2 decimal places
```

(i) Work through the algorithm starting with $N = 16$. Record the values of R and S each time they change and show the value of the output. **[2]**

(ii) Work through the algorithm starting with $N = 2$. Record the values of R and S each time they change and show the value of the output. **[2]**

(iii) What does the algorithm achieve for positive inputs? **[1]**

(iv) Show that the algorithm fails when it is applied to $N = -4$. **[1]**

(v) Describe what happens when the algorithm is applied to $N = -2$. Suggest how the algorithm could be improved to avoid this problem, without imposing a restriction on the allowable input values. **[2]**

3 A *simple* graph is one in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself.

A *connected* graph is one in which every vertex is joined, directly or indirectly, to every other vertex.

A *simply connected* graph is one that is both simple and connected.

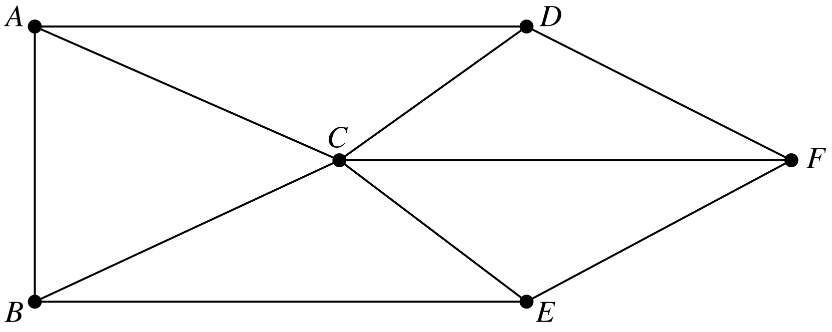
(i) Explain why it is impossible to draw a graph with exactly five vertices of orders 1, 2, 3, 4 and 5. **[1]**

(ii) Explain why there is no simply connected graph with exactly five vertices of orders 2, 2, 3, 4 and 5. State which of the properties 'simple' and 'connected' cannot be achieved. **[2]**

(iii) Calculate the number of arcs in a simply connected graph with exactly five vertices of orders 1, 1, 2, 2 and 4. Hence explain why such a graph cannot be a tree. **[2]**

(iv) Draw a simply connected semi-Eulerian graph with exactly five vertices that is also a tree. By considering the orders of the vertices, explain why it is impossible to draw a simply connected Eulerian graph with exactly five vertices that is also a tree. **[2]**

In the graph below the vertices represent buildings and the arcs represent pathways between those buildings.



(v) By considering the orders of the vertices, explain why it is impossible to walk along these pathways in a continuous route that uses every arc once and only once. Write down the minimum number of arcs that would need to be travelled twice to walk in a continuous route that uses every arc at least once. **[2]**

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4 Consider the following LP problem.

$$\begin{array}{ll} \text{Maximise} & P = -3w + 5x - 7y + 2z, \\ \text{subject to} & w + 2x - 2y - z \leqslant 10, \\ & 2w + 3y - 4z \leqslant 12, \\ & 4w + 5x + y \leqslant 30, \\ \text{and} & w \geqslant 0,\ x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0. \end{array}$$

(i) Represent the problem as an initial Simplex tableau. Explain why the pivot can only be chosen from the x column. **[4]**

(ii) Perform **one** iteration of the Simplex algorithm. Show how each row was obtained and write down the values of w, x, y, z and P at this stage. **[4]**

(iii) Perform a second iteration of the Simplex algorithm. Write down the values of w, x, y, z and P at this stage and explain how you can tell from this tableau that P can be increased without limit. How could you have known from the LP formulation above that P could be increased without limit? **[5]**

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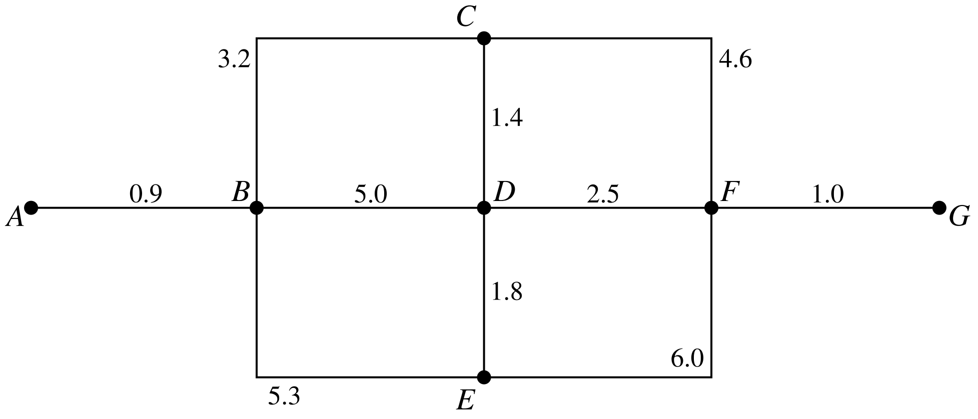
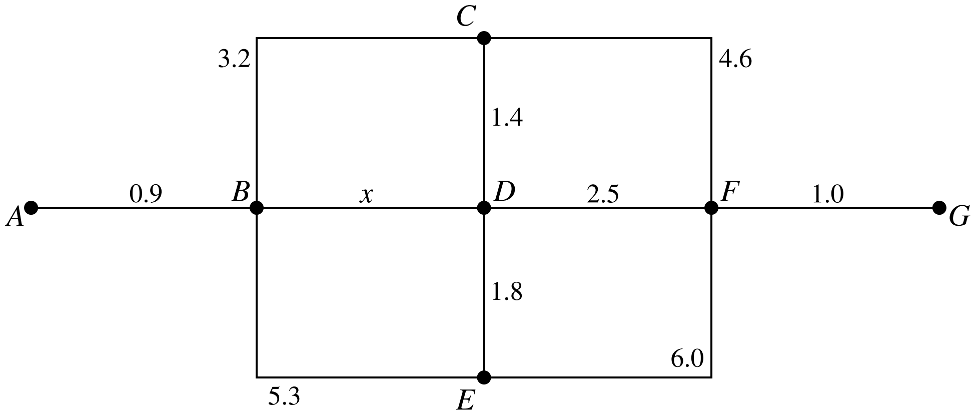
5 The arcs in the network below represent the tracks in a forest and the weights on the arcs represent distances in km.



Dijkstra's algorithm is to be used to find the shortest path from A to G.

(i) Apply Dijkstra's algorithm to find the shortest path from A to G. Show your working, including temporary labels, permanent labels and the order in which permanent labels are assigned. Do not cross out your working values. Write down the route of the shortest path from A to G and give its length. **[6]**

The track joining B and D is washed away in a flood. It is replaced by a new track of unknown length, x km.



(ii) What is the smallest value that x can take so that the route found in part **(i)** is still a shortest path? If the value of x is smaller than this, what is the weight of the shortest path from A to G? **[2]**

(iii) **(a)** For what values of x will vertex E have two temporary labels? Write down the values of these temporary labels. **[2]**

(b) For what values of x will vertex C have two temporary labels? Write down the values of these temporary labels. **[2]**

Dijkstra's algorithm has quadratic order.

(iv) If a computer takes 20 seconds to apply Dijkstra's algorithm to a complete network with 50 vertices, approximately how long will it take for a complete network with 100 vertices? **[2]**

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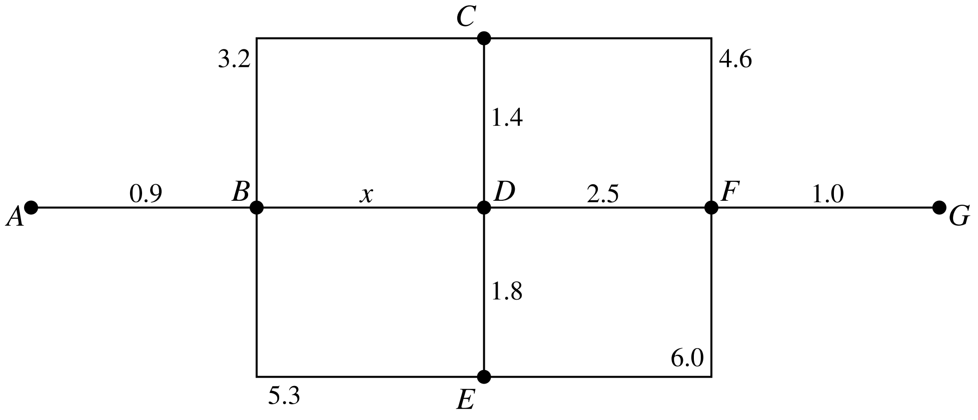
6 The arcs in the network represent the tracks in a forest. The weights on the arcs represent distances in km.



Richard wants to walk along every track in the forest. The total weight of the arcs is $26.7 + x$.

(i) Find, in terms of x, the length of the shortest route that Richard could use to walk along every track, starting at A and ending at G. Show all of your working. **[3]**

(ii) Now suppose that Richard wants to find the length of the shortest route that he could use to walk along every track, starting and ending at A. Show that for $x \leqslant 1.8$ this route has length $(32.4 + 2x)$ km, and for $x \geqslant 1.8$ it has length $(34.2 + x)$ km. **[8]**

Whenever two tracks join there is an information board for visitors to the forest. Shauna wants to check that the information boards have not been vandalised. She wants to find the length of the shortest possible route that starts and ends at A, passing through every vertex at least once.

Consider first the case when x is less than 3.2.

(iii) (a) Apply Prim's algorithm to the network, starting from vertex A, to find a minimum spanning tree. Draw the minimum spanning tree and state its total weight. Explain why the solution to Shauna's problem must be longer than this. **[3]**

(b) Use the nearest neighbour strategy, starting from vertex A, and show that it stalls before it has visited every vertex. **[2]**

Now consider the case when x is greater than 3.2 but less than 4.6.

(iv) (a) Draw the minimum spanning tree and state its total weight. **[2]**

(b) Use the nearest neighbour strategy, starting from vertex A, to find a route from A to G passing through each vertex once. Write down the route obtained and its total weight. Show how a shortcut can give a shorter route from A to G passing through each vertex. Hence, explaining your method, find an upper bound for Shauna's problem. **[4]**